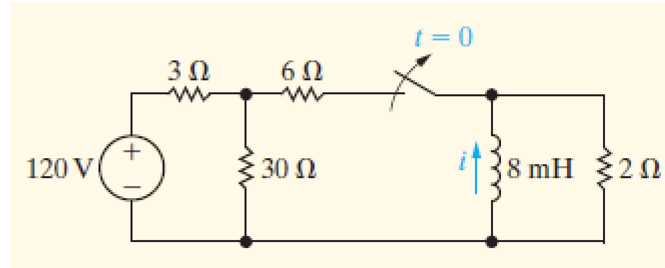
Mathcad Solutions to Assessment Problems from Nilsson and Riedel
Electric Circuits 10th edition, © 2021 R. Doering.
Chapter 7

$$\|(a,b) := \frac{a \cdot b}{a+b}$$

AP 7.1 The switch in the circuit shown has been closed for a long time and is opened at t = 0.
a) Calculate the initial value of i.
b) Calculate the initial energy stored in the inductor.
c) What is the time constant of the circuit for t > 0?
d) What is the numerical expression for i(t) for t > 0?
e) What percentage of the initial energy stored has been dissipated in the 2 Ω resistor 5 ms after the switch has been opened?



$L := 8mH \qquad R := 2\Omega$

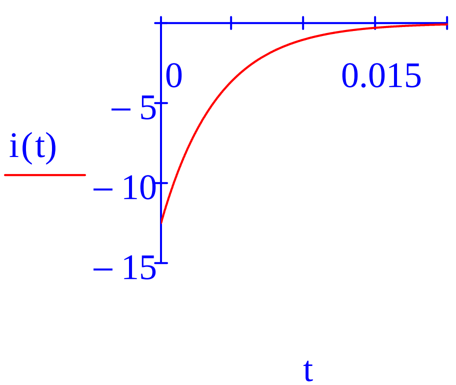
a) Thevenin eq. circuit for source has $V_{th} := \frac{30}{33} \cdot 120V = 109.091\,V \quad R_{th} := (30 \| 3 + 6)\Omega = 8.727\,\Omega$

so initial current is given by $i_0 := \frac{V_{th}}{R_{th}} = 12.5\,A$ b) $w_0 := \frac{1}{2} L \cdot i_0^{\,2} = 625 \cdot mJ$

$mJ \equiv 10^{-3} J$

c) $\tau := \frac{L}{R} = 4 \cdot ms$ d) $i(t) := -i_0 \cdot e^{\frac{-t}{\tau}}$

$$i(t) \rightarrow -12.5 \cdot A \cdot e^{-\frac{250.0 \cdot t}{s}}$$



$$w_5 := \frac{1}{2} L \cdot i(5ms)^2 = 51.303 \cdot mJ$$

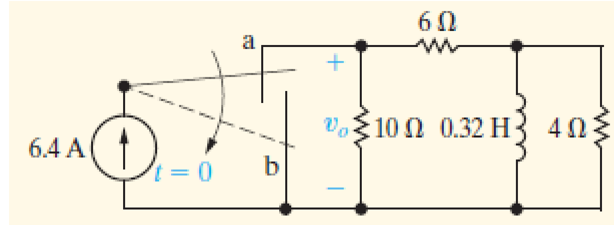
e) energy dissipated at 5ms:

$$1 - \frac{w_5}{w_0} = 91.8 \cdot \%$$

AP 7.2 At t = 0, the switch in the circuit shown moves instantaneously from position a to position b.

a) Calculate v_o for $t > 0^+$.

b) What percentage of the initial energy stored in the inductor is eventually dissipated in the 4 Ω, resistor?



$R := (16 \parallel 4)\Omega = 3.2\,\Omega \quad L := 0.32H \qquad \tau := \frac{L}{R} = 0.1\,s$

a)

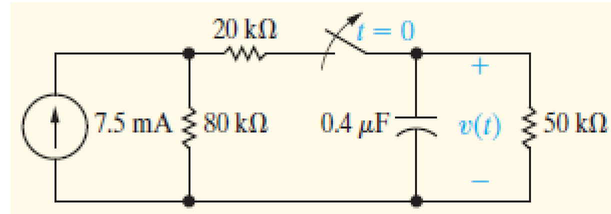
$$i_0 := 6.4A\cdot\frac{(10 \parallel 6)}{6} = 4\,A \qquad i_L(t) := i_0\cdot e^{\frac{-t}{\tau}} \qquad v_o(t) := -i_L(t)\cdot R\cdot\frac{10\Omega}{16\Omega}$$

$$fix := \frac{V}{A\cdot\Omega}$$

$v_o(t) \rightarrow -8.0\cdot A\cdot\Omega\cdot e^{-\frac{10.0\cdot t}{s}}$ for $t \geq 0s$

b) $\frac{16\Omega}{16\Omega + 4\Omega} = 80\cdot\%$

AP 7.3 The switch in the circuit shown has been closed for a long time and is opened at t = 0. Find
a) the initial value of v(t),
b) the time constant for t > 0,
c) the numerical expression for v(t) after the switch has been opened,
d) the initial energy stored in the capacitor, and
e) the length of time required to dissipate 75% of the initially stored energy.



$R := 50k\Omega \qquad C_1 := 0.4\mu F$

a) $v_0 := \frac{50}{70}\cdot(70 \parallel 80)k\Omega\cdot 7.5mA = 200\,V$

b) $\tau := R\cdot C_1 = 0.02\,s$

c) $v(t) := v_0\cdot e^{\frac{-t}{\tau}} \qquad v(t) \rightarrow 200\cdot V\cdot e^{-\frac{50.0\cdot t}{s}}$

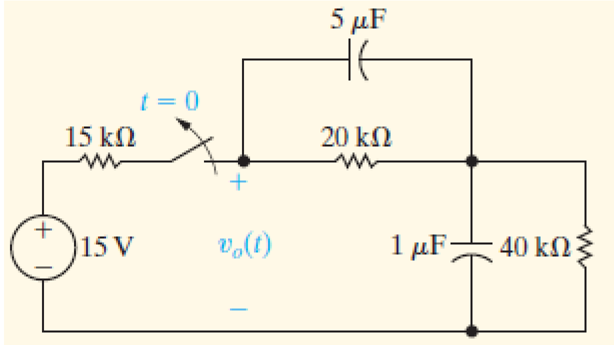
d) $w_0 := \frac{1}{2}C_1\cdot {v_0}^2 = 8\cdot mJ$

e) $t_{75} := -\ln\left(\sqrt{25\%}\right)\cdot\tau = 13.86\cdot ms \qquad w_{75} := \frac{1}{2}\cdot C_1\cdot v\left(t_{75}\right)^2 = 2\cdot mJ$

AP 7.4 The switch in the circuit shown has been closed for a long time before being opened at t = 0.

a) Find $v_o(t)$ for t > 0.

b) What percentage of the initial energy stored in the circuit has been dissipated after the switch has been open for 60 ms?



a)

$$v_1 := \frac{20}{15+20+40}\cdot 15V = 4\,V \qquad v_2 := \frac{40}{15+20+40}\cdot 15V = 8\,V$$

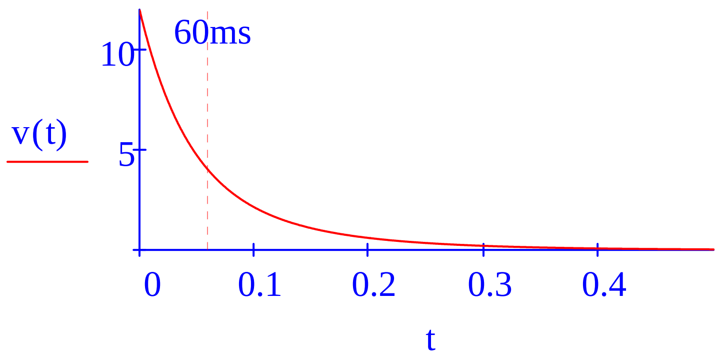
$$\tau_1 := 20k\Omega\cdot 5\mu F = 0.1\,s \qquad \tau_2 := 40k\Omega\cdot 1\mu F = 0.04\,s$$

$$v(t) := v_1\cdot e^{\frac{-t}{\tau_1}} + v_2\cdot e^{\frac{-t}{\tau_2}}$$

$$v(t) \rightarrow 8\cdot V\cdot e^{-\frac{25.0\cdot t}{s}} + 4\cdot V\cdot e^{-\frac{10.0\cdot t}{s}}$$

b)

$$w_0 := \frac{1}{2}5\mu F\cdot {v_1}^2 + \frac{1}{2}\cdot 1\mu F\cdot {v_2}^2 = 72\cdot\mu J$$



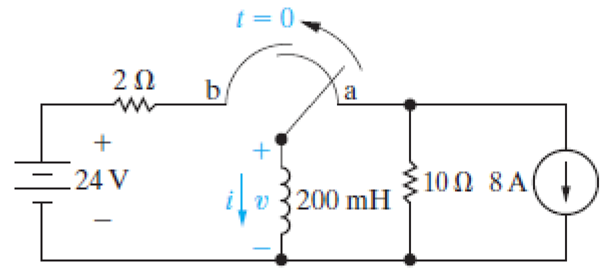
$$w_{60} := \frac{1}{2}5\mu F\cdot\left(v_1\cdot e^{\frac{-60ms}{\tau_1}}\right)^2 + \frac{1}{2}\cdot 1\mu F\cdot\left(v_2\cdot e^{\frac{-60ms}{\tau_2}}\right)^2$$

$$\mu J \equiv 10^{-6} J$$

$$w_{60} = 13.641\cdot\mu J$$

$$1 - \frac{w_{60}}{w_0} = 81.05\cdot\%$$

AP 7.5 Assume that the switch in the circuit shown in Fig. 7.19 has been in position b for a long time, and at t = 0 it moves to position a. Find (a) $i(0^+)$; (b) $v(0^+)$; (c) τ, t > 0; (d) i(t), t > 0; and (e) v(t), $t > 0^+$.



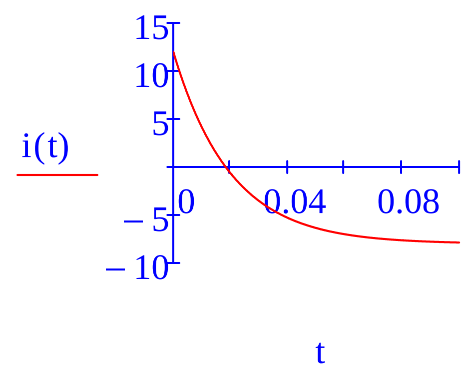
a) $i_0 := \frac{24V}{2\Omega} = 12\,A$ $\qquad i_f := -8A$

b) $v_0 := -(i_0 + 8A)\cdot 10\Omega = -200\,V$

c) $\tau := \frac{200mH}{10\Omega} = 20\cdot ms \qquad \frac{1}{\tau} = 50\,\frac{1}{s}$

d) $i(t) := (i_0 - i_f)e^{\frac{-t}{\tau}} + i_f \qquad i(t) := \left(20e^{\frac{-50t}{s}} - 8\right)\cdot A$

e) $v(t) := 200mH\left(\frac{d}{dt}i(t)\right)$

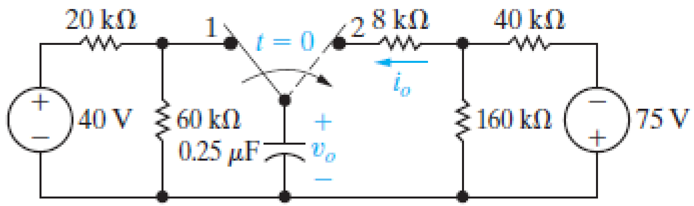


$$v(t) \rightarrow -\frac{200000\cdot A\cdot mH\cdot e^{-\frac{50\cdot t}{s}}}{s} = -200e^{-50\frac{t}{s}}V$$

$$\frac{A\cdot mH}{s} = 1\times 10^{-3}V$$

AP 7.6 a) Find the expression for the voltage across the 160 kΩ resistor in the circuit shown in Fig. 7.22. Let this voltage be denoted v_A, and assume that the reference polarity for the voltage is positive at the upper terminal of the 160 kΩ resistor.

b) Specify the interval of time for which the expression obtained in (a) is valid.



a)

$$v_{A0} := -75V\cdot\frac{160}{200} = -60\,V$$

$$v_{o0} := 40V\cdot\frac{60}{80} = 30\,V$$

using superposition, short out the 75V supply, leaving

$R := (8 + 40 \parallel 160)k\Omega = 40\cdot k\Omega$ so $\tau := R\cdot 0.25\mu F = 0.01\,s$

$$v_o(t) := \left(v_{o0} - v_{A0}\right)e^{\frac{-t}{\tau}} + v_{A0} \qquad v_o(t) \rightarrow 90\cdot V\cdot e^{-\frac{100.0\cdot t}{s}} - 60\cdot V$$

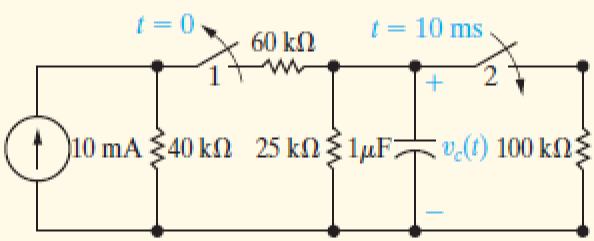
$$v_A(t) := \left(v_{o0} - v_{A0}\right)\cdot e^{\frac{-t}{\tau}}\cdot\frac{(160 \parallel 40)}{(160 \parallel 40) + 8} + v_{A0} \qquad v_A(t) \rightarrow 72\cdot V\cdot e^{-\frac{100.0\cdot t}{s}} - 60\cdot V$$

b) for t > 0

AP 7.7

In the circuit shown, switch 1 has been closed and switch 2 has been open for a long time. At $t = 0$, switch 1 is opened. Then 10 ms later, switch 2 is closed. Find

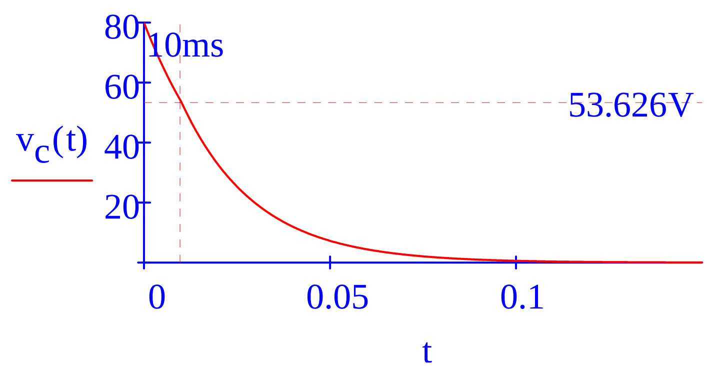
a) $v_c(t)$ for $0 \le t \le 0.01$ s,
b) $v_c(t)$ for $t \ge 0.01$ s,
c) the total energy dissipated in the 25 kΩ resistor, and
d) the total energy dissipated in the 100 kΩ resistor.



$$v_{c0} := \frac{25}{25 + 60 + 40} \cdot 10\text{mA} \cdot 40\text{k}\Omega = 80\,\text{V}$$

$$\frac{1}{25\text{k}\Omega \cdot 1\mu\text{F}} = 40\,\frac{1}{\text{s}} \qquad \frac{1}{(25 \,\|\, 100)\text{k}\Omega \cdot 1\mu\text{F}} = 50\,\frac{1}{\text{s}}$$

$$v_c(t) := \begin{cases} v_{c0} & \text{if } t < 0 \\ v_{c0} \cdot e^{\frac{-40t}{s}} & \text{if } 0 \le t \le 10\text{ms} \quad \text{a)} \\ 53.626 e^{\frac{-50(t-10\text{ms})}{s}}\,\text{V} & \text{otherwise} \quad \text{b)} \end{cases}$$



$$w_0 := \frac{1}{2} \cdot \mu\text{F} \cdot v_{c0}^2 = 3.2 \cdot \text{mJ}$$

$$w_{10} := \frac{1}{2}\mu\text{F} \cdot v_c(10\text{ms})^2 = 1.438 \cdot \text{mJ}$$

c) $$w_{25} := (w_0 - w_{10}) + \left(\frac{100}{125}\right) \cdot w_{10} = 2.912 \cdot \text{mJ}$$

$$\int_{0s}^{200ms} \frac{v_c(t)^2}{25\text{k}\Omega}\,dt = 2.912 \cdot \text{mJ}$$

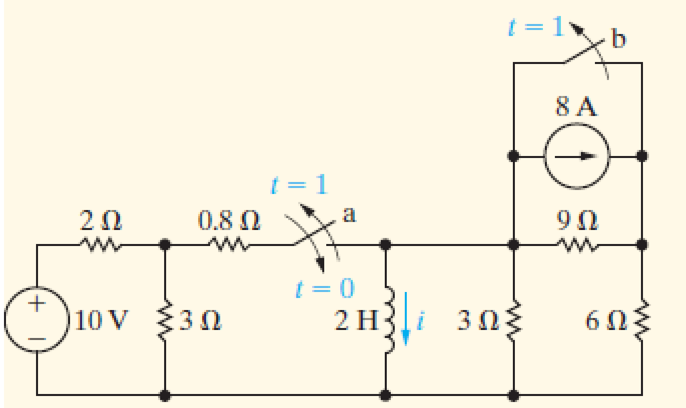
d) $$w_{100} := \frac{25}{125} \cdot w_{10} = 0.288 \cdot \text{mJ}$$

$$\int_{10ms}^{200ms} \frac{v_c(t)^2}{100\text{k}\Omega}\,dt = 0.288 \cdot \text{mJ}$$

$$w_{25} + w_{100} = 3.2 \cdot \text{mJ}$$

AP 7.8

Switch a in the circuit shown has been open for a long time, and switch b has been closed for a long time. Switch a is closed at $t = 0$ and, after remaining closed for 1 s, is opened again. Switch b is opened simultaneously, and both switches remain open indefinitely. Determine the expression for the inductor current i that is valid when (a) $0 \le t \le 1$ s and (b) $t \ge 1$ s.



$$6 \parallel 3 = 2 \qquad 3 \parallel 2 + 0.8 = 2$$

$$\tau_1 := \frac{2H}{1\Omega} = 2\,s \qquad \frac{1}{\tau_1} = 0.5\,\frac{1}{s}$$

$$V_{Th} := \frac{3}{3+2}\cdot 10V = 6\,V \qquad i_{f1} := \frac{V_{Th}}{2\Omega} = 3\,A$$

$$i_{2s} := \left(1 - e^{\frac{-1}{2}}\right)\cdot i_{f1} = 1.18\,A$$

$$V_{Th} := 8A\cdot 9\Omega = 72\,V$$

$$R_{Th} := (9+6)\Omega = 15\,\Omega \qquad i_{2f} := \frac{-V_{Th}}{R_{Th}} = -4.8\,A$$

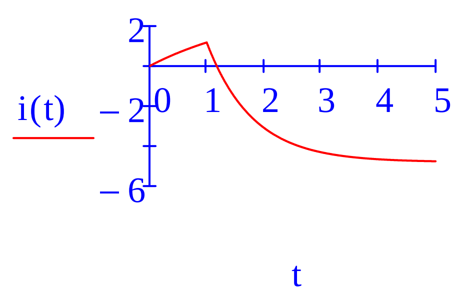
$$\tau_2 := \frac{2H}{R_{Th} \parallel (3\Omega)} = 0.8\,s \qquad \frac{1}{\tau_2} = 1.25\,\frac{1}{s}$$

a) $$i_1(t) := \left(3 - 3e^{\frac{-0.5t}{s}}\right)A$$

$$i(t) := \begin{cases} i_{f1}\cdot\left(1 - e^{\frac{-0.5\cdot t}{s}}\right) & \text{if } 0 \le t \le 1s \\ \left[i_{2f} + \left(i_{2s} - i_{2f}\right)\cdot e^{\frac{-1.25(t-1s)}{s}}\right] & \text{if } t > 1s \end{cases}$$

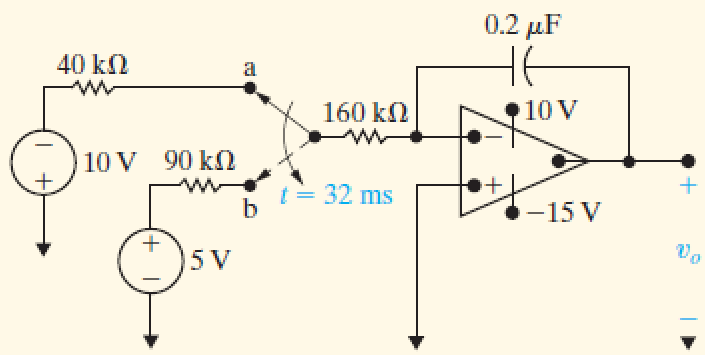
$$i_{2s} - i_{2f} = 5.98\,A$$

b) $$i_2(t) := \left[-4.8 + 5.98\cdot e^{\frac{-1.25(t-1s)}{s}}\right]A$$



AP 7.9

7.9 There is no energy stored in the capacitor at the time the switch in the circuit makes contact with terminal a. The switch remains at position a for 32 ms and then moves instantaneously to position b. How many milliseconds after making contact with terminal a does the op amp saturate?



$$i_1 := \frac{-10V}{(40 + 160)k\Omega} = -50 \cdot \mu A$$

$$v_{32} := \frac{32ms \cdot -i_1}{0.2\mu F} = 8\,V$$

$$i_2 := \frac{5V}{(90 + 160)k\Omega} = 20 \cdot \mu A \qquad \Delta t := ms$$

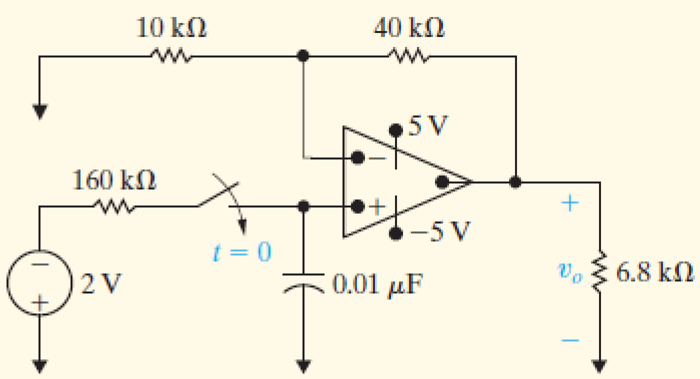
Given $\quad i_2 \cdot \Delta t = (8V + 15V) \cdot 0.2\mu F \qquad \text{Find}(\Delta t) = 230 \cdot ms$

$$t := 32ms + 230ms = 262 \cdot ms$$

AP 7.10

7.10 a) When the switch closes in the circuit shown, there is no energy stored in the capacitor. How long does it take to saturate the op amp?

b) Repeat (a) with an initial voltage on the capacitor of 1 V, positive at the upper terminal.



non-inverting configuration implies gain of 5

so saturation will occur when input voltage hits -1V

$$\tau := 160\text{k}\Omega \cdot 0.01\mu\text{F} = 1.6\cdot\text{ms}$$

$$i_{in}(t) := \frac{-2\text{V}}{160\text{k}\Omega}\cdot e^{\frac{-t}{1.6\text{ms}}}$$

voltage is decaying to half of eventual so

a) $-\ln(.5)\cdot\tau = 1.109\cdot\text{ms}$

verify: $$\frac{1}{0.01\mu\text{F}}\cdot\int_0^{1.109\text{ms}} i_{in}(t)\,dt = -1\text{ V}$$

voltage is decaying to 1/3 of eventual so

$$i_{inb}(t) := \frac{-3\text{V}}{160\text{k}\Omega}\cdot e^{\frac{-t}{\tau}}$$

b) $-\ln\left(\frac{1}{3}\right)\cdot\tau = 1.758\cdot\text{ms}$

verify: $$\frac{1}{0.01\mu\text{F}}\cdot\int_0^{1.758\text{ms}} i_{inb}(t)\,dt + 1\text{V} = -1\text{ V}$$