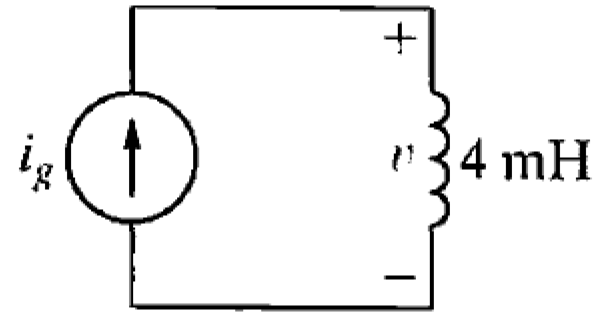
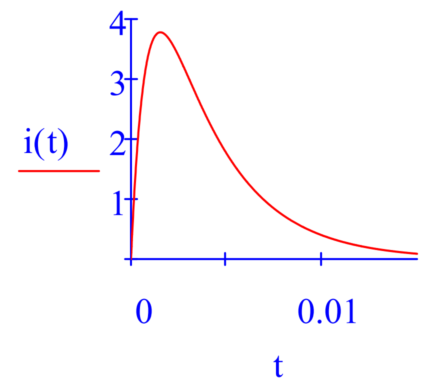
AP 6.1 The current source in the circuit shown generates the current pulse (for t>0)



$$i(t) := \left(8e^{-300\frac{t}{s}} - 8e^{-1200\frac{t}{s}}\right)A$$

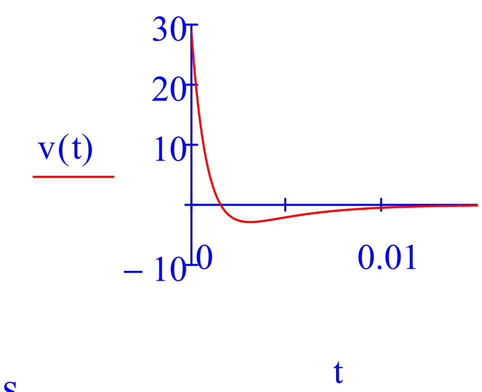
Find (a) v(0); (b) the instant of time, greater than zero, when the voltage v passes through zero; (c) the expression for the power delivered to the inductor; (d) the instant when the power delivered to the inductor is maximum; (e) the maximum power; (f) the instant of time when the stored energy is maximum; and (g) the maximum energy stored in the inductor.



a)

$$v(t) := 4mH\cdot\frac{d}{dt}i(t) \rightarrow -4\cdot A\cdot mH\cdot\left(\frac{2400\cdot e^{-\frac{300\cdot t}{s}}}{s} - \frac{9600\cdot e^{-\frac{1200\cdot t}{s}}}{s}\right)$$

$$v(0) = 28.8\,V$$



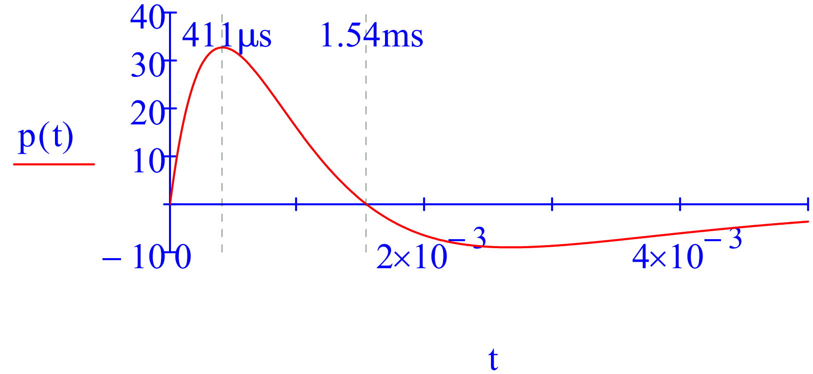
b) Given $\quad t_{zv} := 1ms \qquad v(t_{zv}) = 0 \qquad t_{zv} := Find(t_{zv}) = 1.54\cdot ms$

c) $p(t) := v(t) \cdot i(t) \text{ expand } \rightarrow \dfrac{384000 \cdot A^2 \cdot mH \cdot e^{-\frac{1500 \cdot t}{s}}}{s} - \dfrac{76800 \cdot A^2 \cdot mH \cdot e^{-\frac{600 \cdot t}{s}}}{s} - \dfrac{307200 \cdot A^2 \cdot mH \cdot e^{-\frac{2400 \cdot t}{s}}}{s}$

$$\frac{A^2 \cdot mH}{s} = 1 \cdot mW$$

$$p(t) := \left(384 \cdot e^{-\frac{1500 \cdot t}{s}} - 76.8 \cdot e^{-\frac{600 \cdot t}{s}} - 307.2 \cdot e^{-\frac{2400 \cdot t}{s}}\right) W$$

while t>0



d) Given $t_{pm} := 0s$ $\quad \dfrac{d}{dt_{pm}} p(t_{pm}) = 0 \quad t_{pmax} := \text{Find}(t_{pm}) = 411.051 \cdot \mu s$

e) $p(t_{pmax}) = 32.719\, W$

f) Given $t_{em} := 1ms \quad p(t_{em}) = 0 \quad t_{emax} := \text{Find}(t_{em}) = 1.54 \cdot ms$

g) $\displaystyle\int_0^{t_{emax}} p(t)\, dt = 28.573 \cdot mJ$

AP 6.2 The voltage at the terminals of a 0.6 μF capacitor is 0 for t < 0 and

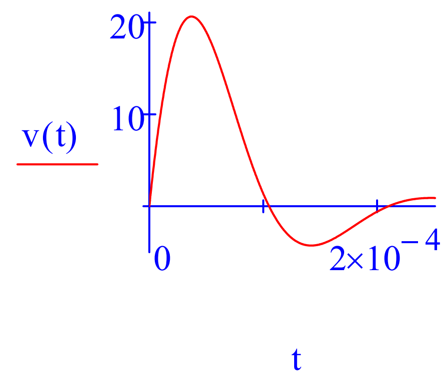
$$v(t) := 40 \cdot e^{-15000 \frac{t}{s}} \cdot \sin\left(30000 \frac{t}{s}\right) V$$

Find

(a) i(0);

(b) the power delivered to the capacitor at t = π/80 ms;

(c) the energy stored in the capacitor at t = π/80 ms.



a)

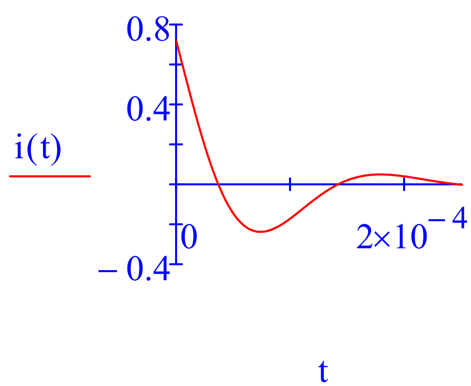
$$i(t) := 0.6\mu F \cdot \frac{d}{dt} v(t) \rightarrow 0.6 \cdot \mu F \cdot \left(\frac{1200000 \cdot V \cdot \cos\left(\frac{30000 \cdot t}{s}\right) \cdot e^{-\frac{15000 \cdot t}{s}}}{s} - \frac{600000 \cdot V \cdot \sin\left(\frac{30000 \cdot t}{s}\right) \cdot e^{-\frac{15000 \cdot t}{s}}}{s} \right)$$

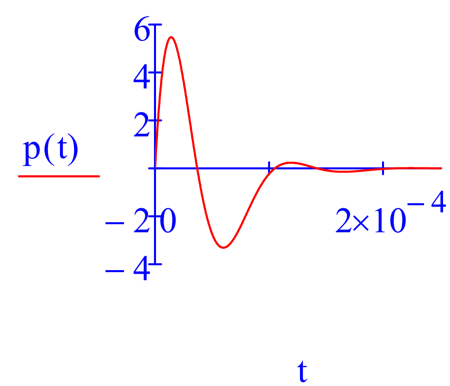
$$i(0) = 0.72 \cdot A$$

b) $p(t) := v(t) \cdot i(t)$

$$p\left(\frac{\pi}{80} ms\right) = -649.2 \cdot mW$$

c) $$\int_0^{\frac{\pi}{80} ms} p(t)\, dt = 126.13 \cdot \mu J$$





AP 6.3 The current in a 0.6μF capacitor is 0 for t < 0 and 3 cos 50,000t A for t > 0.

$C_1 := 0.6\mu F$

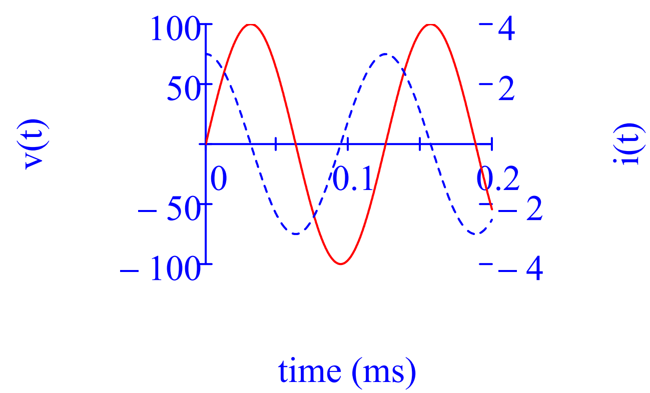
$i(t) := 3\cos\left(50000\frac{t}{s}\right)A$ for $t \ge 0$

Find (a) v(t)
(b) the maximum power delivered to the capacitor at any one instant of time; and
(c) the maximum energy stored in the capacitor at any one instant of time.

a) $\int 3\cos(500t)\,dt \rightarrow \frac{3\cdot\sin(500\cdot t)}{500}$

$v(t) := \frac{1}{C_1}\cdot\frac{3}{50000}\cdot\sin\left(50000\frac{t}{s}\right)\cdot A\cdot s$

$v(0) = 0\,V$



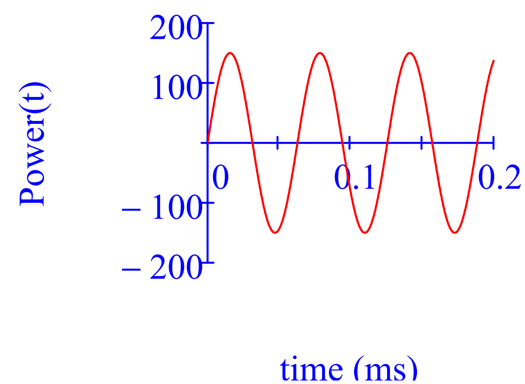
b) $p(t) := i(t)\cdot v(t)$

$p(t)\ \text{simplify} \rightarrow \frac{0.00015\cdot A^2\cdot s\cdot\sin\left(\frac{100000.0\cdot t}{s}\right)}{\mu F}$

$\frac{A^2 s}{F} = 1\,W$

$p(t) := 150\sin\left(100000\frac{t}{s}\right)W$

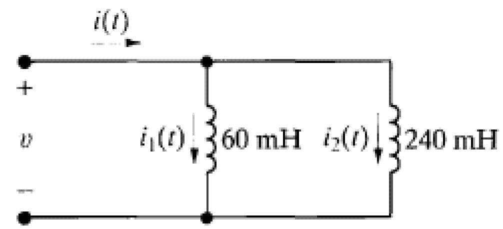
$p_{max} := 150W$



c) $energy(t) := \int_0^t p(\tau)\,d\tau$

$energy\left(\frac{\pi}{100000}s\right) = 3\cdot mJ$

AP 6.4 The initial values of i_1 and i_2 in the circuit shown are + 3 A and - 5 A, respectively. The voltage at the terminals of the parallel inductors for t > 0 is



$$v(t) := -30e^{-50\frac{t}{s}}\,mV$$

a) If the parallel inductors are replaced by a single inductor, what is its inductance?
b) What is the initial current and its reference direction in the equivalent inductor?
c) Use the equivalent inductor to find i(t).
d) Find $i_1(t)$ and $i_2(t)$. Verify that the solutions for $i_1(t)$, $i_2(t)$, and i(t) satisfy Kirchhoff's current law.

a) $(60mH) \parallel (240mH) = 48 \cdot mH$

b) $-2A$ Downward

c) $$\int v(t)\,dt \rightarrow \frac{3 \cdot mV \cdot s \cdot e^{-\frac{50 \cdot t}{s}}}{5}$$

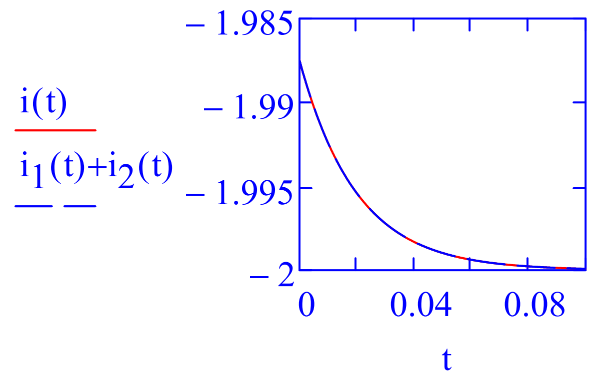
$$\frac{mV \cdot s}{mH} = 1\,A$$

$$i(t) := \frac{1}{48} \cdot \frac{3}{5}\,e^{-50\frac{t}{s}}\,A + -2A$$

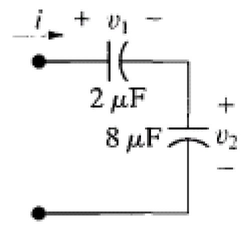
d) $$i_1(t) := \frac{1}{60} \cdot \frac{3}{5}\,e^{-50\frac{t}{s}}\,A + 3A$$

$$i_2(t) := \frac{1}{240} \cdot \frac{3}{5}\,e^{-50\frac{t}{s}}\,A - 5A$$

$i_1(t) + i_2(t) - i(t) \text{ simplify } \rightarrow 0$



AP 6.5 The current at the terminals of the two capacitors shown is



$$i(t) := 240e^{-10\frac{t}{s}}\mu A \quad \text{for} \quad t \geq 0$$

The initial values of v_1 and v_2 are -10V and -5V, respectively. Calculate the total energy trapped in the capacitors as $t \rightarrow \infty$ (Hint: Don't combine the capacitors in series—find the energy trapped in each, and then add.)

$$q_{inf} := \int_0^{\infty \cdot s} i(t)\, dt = 24 \cdot \mu C$$

$$q_1 := 2\mu F \cdot -10V + q_{inf} = 4 \cdot \mu C$$

$$q_2 := 8\mu F \cdot -5V + q_{inf} = -16 \cdot \mu C$$

$$v_{1inf} := \frac{q_1}{2\mu F} = 2\,V$$

$$v_{2inf} := \frac{q_2}{8\mu F} = -2\,V$$

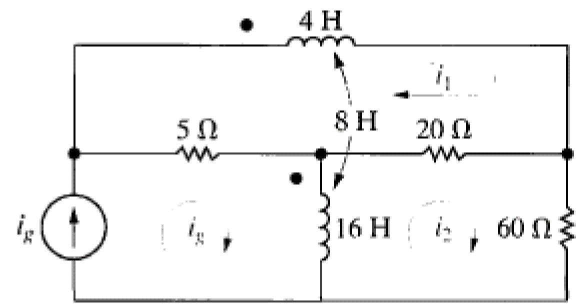
$$energy_1 := \frac{1}{2} \cdot 2\mu F \cdot v_{1inf}^{\,2} = 4 \cdot \mu J$$

$$energy_2 := \frac{1}{2} \cdot 8\mu F \cdot v_{2inf}^{\,2} = 16 \cdot \mu J$$

$$energy := energy_1 + energy_2 = 20 \cdot \mu J$$

Note that the initial conditions for this problem are impractical as the two capacitors have unequal charges.

AP 6.6 a) Write a set of mesh-current equations for the circuit in Example 6.6. If the dot on the 4H inductor is at the right-hand terminal, the reference direction of i_g, is reversed, and the 60 Ω resistor is increased to 780 Ω.
b) Verify that if there is no energy stored in the circuit at t = 0,and if



$$i_g(t) := 1.96\cdot\left(1 - e^{-4\frac{t}{s}}\right)A$$

the solutions to the differential equations derived in (a) of this Assessment Problem are

$$i_1(t) := \left(-0.4 - 11.6e^{-4\frac{t}{s}} + 12e^{-5\frac{t}{s}}\right)A$$

$$i_2(t) := \left(-0.01 - 0.99e^{-4\frac{t}{s}} + e^{-5\frac{t}{s}}\right)A$$

a)

$$4H\cdot\frac{d}{dt}i_1(t) - 8H\cdot\left[\frac{d}{dt}\left(-i_g(t) - i_2(t)\right)\right] + 20\Omega\left(i_1(t) - i_2(t)\right) + 5\Omega\left(i_1(t) + i_g(t)\right) = 0$$

$$20\Omega\cdot\left(i_2(t) - i_1(t)\right) + 780\Omega\cdot i_2(t) + 16H\cdot\frac{d}{dt}\left(i_2(t) + i_g(t)\right) + 8H\cdot\frac{d}{dt}i_1(t) = 0$$

rewriting :

$$4\cdot H\cdot\frac{d}{dt}i_1(t) + 25\cdot\Omega\cdot i_1(t) + 8\cdot H\cdot\frac{d}{dt}i_2(t) - 20\Omega\cdot i_2(t) = -\left(8\cdot H\cdot\frac{d}{dt}i_g(t) + 5\cdot\Omega\cdot i_g(t)\right)$$

$$8H\cdot\frac{d}{dt}i_1(t) - 20\cdot\Omega\cdot i_1(t) + 16\cdot H\cdot\frac{d}{dt}i_2(t) + 800\cdot\Omega\cdot i_2(t) = -16\cdot H\cdot\frac{d}{dt}i_g(t)$$

now check initial and final values.

$i_1(0) = 0\,A$ $\qquad$ $i_2(0) = 0\,A$

$i_1(\infty\cdot s) = -0.4\,A$ $\qquad$ $i_2(\infty\cdot s) = -0.01\,A$

$i_{final} := i_g(\infty\cdot s) = 1.96\,A$ $\qquad$ $v_{final} := (5 \parallel 20 \parallel 780)\Omega\cdot i_{final} = 7.8\,V$

$i_{1final} := \dfrac{-v_{final}}{(20 \parallel 780)\Omega} = -0.4\,A$ $\qquad$ $i_{2final} := \dfrac{-v_{final}}{780\Omega} = -0.01\,A$

$t := 0$

$$4\cdot H\cdot\frac{d}{dt}i_1(t) + 25\cdot\Omega\cdot i_1(t) + 8\cdot H\cdot\frac{d}{dt}i_2(t) - 20\Omega\cdot i_2(t) = -\left(8\cdot H\cdot\frac{d}{dt}i_g(t) + 5\cdot\Omega\cdot i_g(t)\right) \text{ simplify } \rightarrow 4.5438388140$$

$4.5438388140730279993\text{e-}27\cdot A\cdot\Omega = 0\,V$

$$8H\cdot\frac{d}{dt}i_1(t) - 20\cdot\Omega\cdot i_1(t) + 16\cdot H\cdot\frac{d}{dt}i_2(t) + 800\cdot\Omega\cdot i_2(t) = -16\cdot H\cdot\frac{d}{dt}i_g(t) \text{ simplify } \rightarrow 1.6155871338926321775$$

$1.6155871338926321775\text{e-}26\cdot A\cdot\Omega = 0\,V$