Mathcad Solutions to Assesment Problems from Nilsson and Riedel

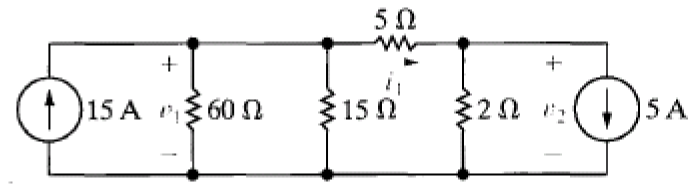
Electric Circuits 9th edition,

Chapter 4

AP 4.1 a) For the circuit shown, use the node-voltage method to find v_1, v_2, and i_1.

b) How much power is delivered to the circuit by the 15 A source?

c) Repeat (b) for the 5 A source.



Assume all currents are leaving nodes. Use dot subcripts because no v_0.

$$v_1 := 0V \qquad v_2 := 0V$$

given

node v_1: $$-15A + \frac{1}{60\Omega}\cdot v_1 + \frac{1}{15\Omega}\cdot v_1 + \frac{1}{5\Omega}\cdot(v_1 - v_2) = 0$$

$$\frac{1}{5\Omega}\cdot(v_2 - v_1) + \frac{1}{2\Omega}\cdot v_2 + 5A = 0$$

$$\text{find}(v_1, v_2) = \begin{pmatrix} 60 \\ 10 \end{pmatrix} V \qquad i_1 := \frac{(60 - 10)V}{5\Omega} = 10\,A$$

in matrix notation:

$$G := \begin{pmatrix} \frac{1}{60} + \frac{1}{15} + \frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{5} + \frac{1}{2} \end{pmatrix}\cdot\frac{1}{\Omega} \qquad i := \begin{pmatrix} 15A \\ -5A \end{pmatrix} \qquad G^{-1}\cdot i = \begin{pmatrix} 60 \\ 10 \end{pmatrix} V$$

Alternate method finding currents in resistors:

assign names to the resistor values: $r := \begin{pmatrix} 60 \\ 5 \\ 15 \\ 2 \end{pmatrix}\Omega$ so $r_0 = 60\,\Omega$ $r_2 = 15\,\Omega$ etc. The currents through the resistors will use the same subscripts.

Write out KCL equations for the top two nodes:

$$i_0 + i_1 + i_2 - 15A = 0$$

$$i_3 - i_1 + 5A = 0$$

Write out two KVL loops for the loops without current sources:

$$r_2 i_2 - r_0 \cdot i_0 = 0$$

$$r_1 i_1 + r_3 i_3 - r_2 i_2 = 0$$

to keep the units consistent, multiply through the KCL equations by 1Ω.

$$i := i$$

now factor these equations into matrix form:

$$\begin{pmatrix} 1\Omega & 1\Omega & 1\Omega & 0 \\ 0 & -1\Omega & 0 & 1\Omega \\ -r_0 & 0 & r_2 & 0 \\ 0 & r_1 & -r_2 & r_3 \end{pmatrix} \cdot \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 15 \\ -5 \\ 0 \\ 0 \end{pmatrix} \cdot V \rightarrow \begin{pmatrix} \Omega \cdot i_0 + \Omega \cdot i_1 + \Omega \cdot i_2 \\ \Omega \cdot i_3 - \Omega \cdot i_1 \\ 15 \cdot \Omega \cdot i_2 - 60 \cdot \Omega \cdot i_0 \\ 5 \cdot \Omega \cdot i_1 - 15 \cdot \Omega \cdot i_2 + 2 \cdot \Omega \cdot i_3 \end{pmatrix} = \begin{pmatrix} 15 \cdot V \\ -5 \cdot V \\ 0 \\ 0 \end{pmatrix}$$

invert the square matrix and multiply through the equation
(after you get the hang of this, you won't nee to take all these steps)

$$\begin{pmatrix} 1\Omega & 1\Omega & 1\Omega & 0 \\ 0 & -1\Omega & 0 & 1\Omega \\ -r_0 & 0 & r_2 & 0 \\ 0 & r_1 & -r_2 & r_3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 1\Omega & 1\Omega & 1\Omega & 0 \\ 0 & -1\Omega & 0 & 1\Omega \\ -r_0 & 0 & r_2 & 0 \\ 0 & r_1 & -r_2 & r_3 \end{pmatrix} \cdot \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 1\Omega & 1\Omega & 1\Omega & 0 \\ 0 & -1\Omega & 0 & 1\Omega \\ -r_0 & 0 & r_2 & 0 \\ 0 & r_1 & -r_2 & r_3 \end{pmatrix}^{-1} \cdot \begin{pmatrix} 15 \\ -5 \\ 0 \\ 0 \end{pmatrix} \cdot V \rightarrow \begin{pmatrix} i_0 \\ i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} \frac{V}{\Omega} \\ \frac{10 \cdot V}{\Omega} \\ \frac{4 \cdot V}{\Omega} \\ \frac{5 \cdot V}{\Omega} \end{pmatrix}$$

assign to variable

$$i := \begin{pmatrix} \frac{V}{\Omega} \\ \frac{10 \cdot V}{\Omega} \\ \frac{4 \cdot V}{\Omega} \\ \frac{5 \cdot V}{\Omega} \end{pmatrix} \qquad i = \begin{pmatrix} 1 \\ 10 \\ 4 \\ 5 \end{pmatrix} A$$

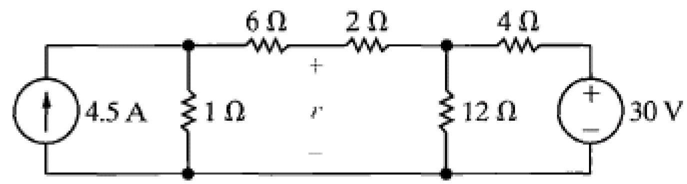
now use the currents to find the voltages:

a) $v_1 := i_0 \cdot r_0 = 60\,V$ $\quad v_2 := i_3 \cdot r_3 = 10\,V$ $\quad i_1 = 10\,A$

b) $P_{15} := 15A \cdot v_1 = 900\,W$

c) $P_5 := -5A \cdot v_2 = -50\,W$

AP 4.2 Use the node-voltage method to find v in the circuit shown.



given

$$v := \begin{pmatrix} 0 \\ 0 \end{pmatrix} V$$

$$-4.5A + \frac{1}{1\Omega}\cdot v_0 + \frac{1}{(6+2)\Omega}\cdot(v_0 - v_1) = 0$$

$$\frac{1}{(6+2)\Omega}\cdot(v_1 - v_0) + \frac{1}{12\Omega}\cdot v_1 + \frac{1}{4\Omega}\cdot(v_1 - 30V) = 0$$

$$v := \text{find}(v) = \begin{pmatrix} 6 \\ 18 \end{pmatrix} V$$

so the voltage at the center point is

$$v_0 - \frac{(v_0 - v_1)}{(6+2)\Omega}\cdot 6\Omega = 15\,V$$

in matrix form:

$$G := \begin{pmatrix} \frac{1}{1} + \frac{1}{6+2} & -\frac{1}{6+2} \\ -\frac{1}{6+2} & \frac{1}{6+2} + \frac{1}{12} + \frac{1}{4} \end{pmatrix}\cdot\frac{1}{\Omega} \qquad i := \begin{pmatrix} 4.5A \\ \frac{30V}{4\Omega} \end{pmatrix} \qquad G^{-1}\cdot i = \begin{pmatrix} 6 \\ 18 \end{pmatrix} V$$

Alternate solution:

$$r := \begin{pmatrix} 1 \\ 6 \\ 2 \\ 12 \\ 4 \end{pmatrix}\Omega$$

label i's to match r's
assume all currents go right or down.

assign one ohm to lower case L.

$l := 1\Omega$ (looks like one. appears in KCL lines in vn.)

This solution is approaching the spice style solution and so doesn't eliminate non-essential nodes! so the node between r1 and r2 is included.

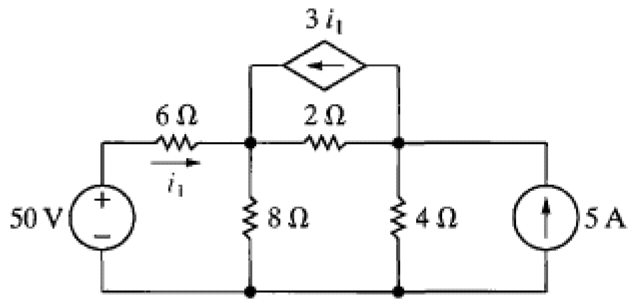
$$vn := \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -r_0 & r_1 & r_2 & r_3 & 0 \\ 0 & 0 & 0 & -r_3 & r_4 \end{pmatrix} \qquad v := \begin{pmatrix} 4.5 \\ 0 \\ 0 \\ 0 \\ -30 \end{pmatrix} V \qquad i := vn^{-1}\cdot v \qquad i = \begin{pmatrix} 6 \\ -1.5 \\ -1.5 \\ 1.5 \\ -3 \end{pmatrix} A$$

so the value of v is $i_2\cdot r_2 + i_3 r_3 = 15\,V$ or $i_0\cdot r_0 - i_1 r_1 = 15\,V$

not asked for here, but because each r has its own i, power is simpler $P := i^2 r = 117\,W$ $\qquad v_4\cdot i_4 + 4.5A\cdot i_0\cdot r_0 = 117\,W$

AP 4.3 a) Use the node-voltage method to find the power associated with each source in the circuit shown,
b) State whether the source is delivering power to the circuit or extracting power from the circuit.

assume all currents go right or down.



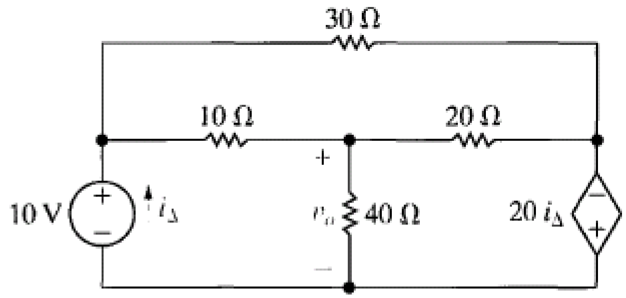
$$r := \begin{pmatrix} 8 \\ 6 \\ 2 \\ 4 \end{pmatrix} \Omega$$

$$vn := \begin{pmatrix} 1 & -1 - 3\Omega & 1 & 0 \\ 0 & 3\Omega & -1 & 1 \\ r_0 & r_1 & 0 & 0 \\ -r_0 & 0 & r_2 & r_3 \end{pmatrix} \qquad v := \begin{pmatrix} 0 \\ 5 \\ 50 \\ 0 \end{pmatrix} V \qquad i := vn^{-1} \cdot v \qquad i = \begin{pmatrix} 4 \\ 3 \\ 8 \\ 4 \end{pmatrix} A$$

a) $P_{50} := -50V \cdot i_1 = -150\,W \qquad P_{3i1} := -3 \cdot i_1 \cdot i_2 \cdot r_2 = -144\,W \qquad P_{5A} := -5A \cdot i_3 \cdot r_3 = -80\,W$

b) All sources are delivering power to the circuit.

AP 4.4 Use the node-voltage method to find v_o in the circuit shown.



$$r := \begin{pmatrix} 40 \\ 10 \\ 30 \\ 20 \end{pmatrix} \Omega$$

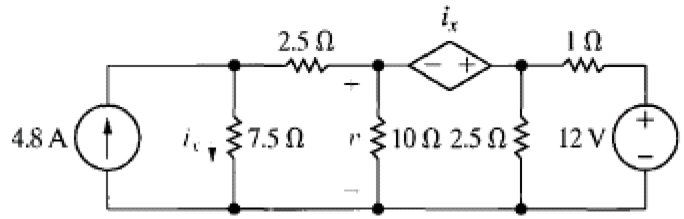
assume each r has a coresponding i.
assume all currents go right or down.

$$i_\Delta := i_1 + i_2$$

$$vn := \begin{pmatrix} 1 & -1 & 0 & 1 \\ 0 & -r_1 & r_2 & -r_3 \\ r_0 & r_1 & 0 & 0 \\ -r_0 & -20\Omega & -20\Omega & r_3 \end{pmatrix} \quad v := \begin{pmatrix} 0 \\ 0 \\ 10 \\ 0 \end{pmatrix} V \quad i := vn^{-1} \cdot v \quad i = \begin{pmatrix} 0.6 \\ -1.4 \\ -1.8 \\ -2 \end{pmatrix} A \quad v_r := \overrightarrow{(i \cdot r)} = \begin{pmatrix} 24 \\ -14 \\ -54 \\ -40 \end{pmatrix} V$$

$$v_{r_0} = 24\,V$$

AP 4.5 Use the node-voltage method to find v in the circuit shown.



use super node across controlled voltage.

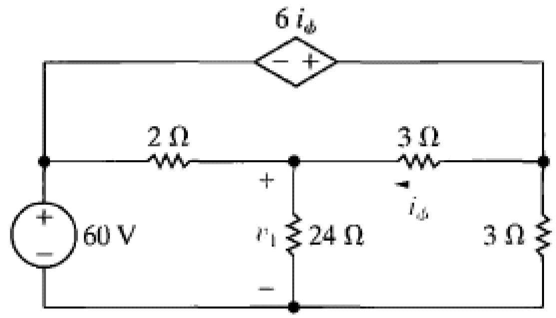
$$r := \begin{pmatrix} 7.5 \\ 2.5 \\ 10 \\ 2.5 \\ 1 \end{pmatrix} \Omega$$

$$vn := \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 \\ -r_0 & r_1 & r_2 & 0 & 0 \\ -1\Omega & 0 & -r_2 & r_3 & 0 \\ 0 & 0 & 0 & -r_3 & r_4 \end{pmatrix} \quad v := \begin{pmatrix} 4.8 \\ 0 \\ 0 \\ 0 \\ -12 \end{pmatrix} V \quad i := vn^{-1} v \quad i = \begin{pmatrix} 2 \\ 2.8 \\ 0.8 \\ 4 \\ -2 \end{pmatrix} A$$

$$vn = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 \\ -7.5 & 2.5 & 10 & 0 & 0 \\ -1 & 0 & -10 & 2.5 & 0 \\ 0 & 0 & 0 & -2.5 & 1 \end{pmatrix} \Omega$$

$$v := i_2 \cdot r_2 = 8\text{ V}$$

AP 4.6 Use the node-voltage method to find v_1 in the circuit shown.



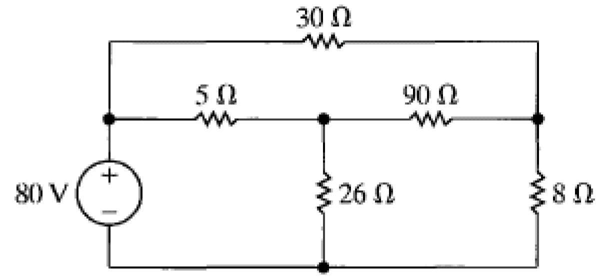
$$r := \begin{pmatrix} 2 \\ 24 \\ 3 \\ 3 \end{pmatrix} \Omega$$

$$vn := \begin{pmatrix} -1 & 1 & 1 & 0 \\ r_0 & r_1 & 0 & 0 \\ -r_0 & 0 & -r_2 + 6\Omega & 0 \\ 0 & -r_1 & r_2 & r_3 \end{pmatrix} \qquad v := \begin{pmatrix} 0 \\ 60 \\ 0 \\ 0 \end{pmatrix} V \qquad i := vn^{-1} \cdot v \qquad i = \begin{pmatrix} 6 \\ 2 \\ 4 \\ 12 \end{pmatrix} A$$

$$v := \overrightarrow{(i \cdot r)} \qquad v_1 = 48\ V$$

AP 4.7

Use the mesh-current method to find (a) the power delivered by the 80 V source to the circuit shown and (b) the power dissipated in the 8 Ω resistor.



$$r := \begin{pmatrix} 5 \\ 26 \\ 30 \\ 90 \\ 8 \end{pmatrix} \Omega$$

$$cm := \begin{pmatrix} r_0 + r_1 & -r_0 & -r_1 \\ -r_0 & r_0 + r_2 + r_3 & -r_3 \\ -r_1 & -r_3 & r_1 + r_3 + r_4 \end{pmatrix}$$

$$v := \begin{pmatrix} 80 \\ 0 \\ 0 \end{pmatrix} V \qquad i := cm^{-1} \cdot v \qquad i = \begin{pmatrix} 5 \\ 2 \\ 2.5 \end{pmatrix} A$$

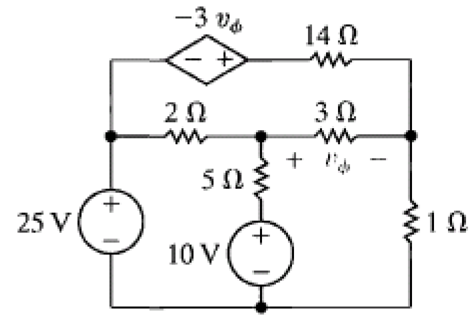
$$i_r := \begin{pmatrix} i_0 - i_1 \\ i_0 - i_2 \\ i_1 \\ i_2 - i_1 \\ i_2 \end{pmatrix} \qquad i_r = \begin{pmatrix} 3 \\ 2.5 \\ 2 \\ 0.5 \\ 2.5 \end{pmatrix} A$$

$$i_r^{\,2} \cdot r = 400\ W$$

$$(-v)_0 \cdot i_0 = -400\ W$$

AP 4.8 a) Determine the number of mesh-current equations needed to solve the circuit shown.

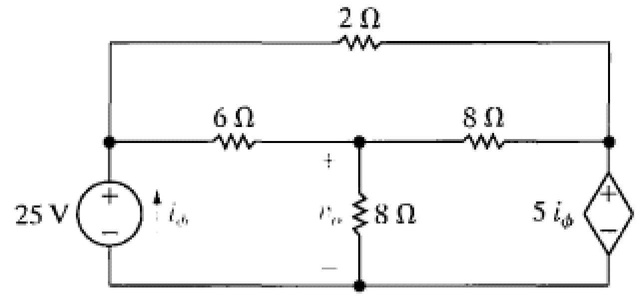
b) Use the mesh-current method to find how much power is being delivered to the dependent voltage source.



$$r := \begin{pmatrix} 2 \\ 5 \\ 14 \\ 3 \\ 1 \end{pmatrix} \cdot \Omega \qquad cm := \begin{bmatrix} r_0 + r_1 & -r_0 & -r_1 \\ -r_0 & (r_2 + r_3 + r_0) - 3\,r_3 & -r_3 + 3 \cdot r_3 \\ -r_1 & -r_3 & r_1 + r_3 + r_4 \end{bmatrix} \qquad v := \begin{pmatrix} 25 - 10 \\ 0 \\ 10 \end{pmatrix} V$$

$$i := cm^{-1} \cdot v \qquad i = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix} A \qquad 3 \cdot (i_2 - i_1) \cdot r_3 \cdot i_1 = -36\,W$$

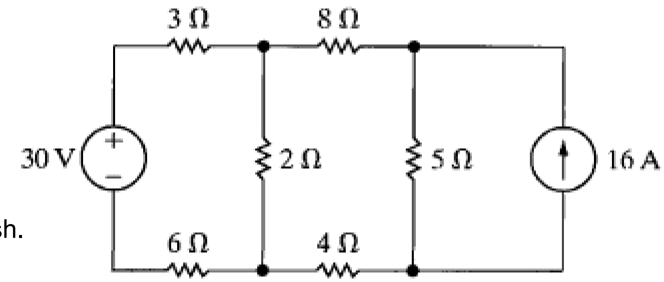
AP 4.9 Use the mesh-current method to find v_o in the circuit shown.



$$r := \begin{pmatrix} 6 \\ 8 \\ 2 \\ 8 \end{pmatrix} \Omega \quad cm := \begin{pmatrix} r_0 + r_1 & -r_0 & -r_1 \\ -r_0 & r_0 + r_2 + r_3 & -r_3 \\ -r_1 + 5 \cdot \Omega & -r_3 & r_1 + r_3 \end{pmatrix} \quad v := \begin{pmatrix} 25 \\ 0 \\ 0 \end{pmatrix} V$$

$$i := cm^{-1} \cdot v \qquad i = \begin{pmatrix} 4 \\ 2.5 \\ 2 \end{pmatrix} A \qquad i_{r1} := (i_0 - i_2) r_1 = 16\,V$$

AP 4.10 Use the mesh-current method to find the power dissipated in the 2 Ω resistor in the circuit shown.



$$r := \begin{pmatrix} 3 \\ 2 \\ 6 \\ 8 \\ 5 \\ 4 \end{pmatrix} \Omega$$

Assume currents i_0 -i_1 run clockwise.

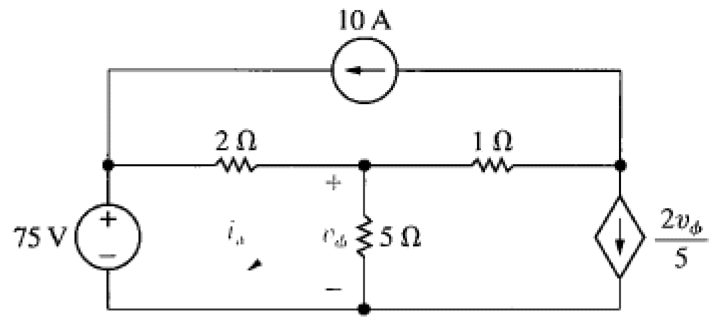
the constant current source needs no mesh.

$$cm := \begin{pmatrix} r_0 + r_1 + r_2 & -r_1 \\ -r_1 & r_1 + r_3 + r_4 + r_5 \end{pmatrix} \quad v := \begin{pmatrix} 30V \\ -16A \cdot r_4 \end{pmatrix} \quad i := cm^{-1} \cdot v = \begin{pmatrix} 2 \\ -4 \end{pmatrix} A \quad (i_0 - i_1)^2 \cdot r_1 = 72\,W$$

alternatively use three loops:

$$cm := \begin{pmatrix} r_0 + r_1 + r_2 & -r_1 & 0 \\ -r_1 & r_1 + r_3 + r_4 + r_5 & -r_4 \\ 0 & 0 & 1\Omega \end{pmatrix} \quad v := \begin{pmatrix} 30 \\ 0 \\ -16 \end{pmatrix} V \quad i := cm^{-1} \cdot v = \begin{pmatrix} 2 \\ -4 \\ -16 \end{pmatrix} A$$

AP 4.11 Use the mesh-current method to find the mesh current i_a in the circuit shown.



$$i_b = \frac{2}{5}\cdot 5\cdot(i_a - i_b) \text{ solve}, i_b \rightarrow \frac{2\cdot i_a}{3}$$

so $\quad 2\cdot i_a - 3\cdot i_b = 0$

$$(i_a + 10A)\cdot 2\Omega + \frac{i_a}{3}\cdot 5\Omega = 75V \text{ solve}, i_a \rightarrow \frac{3\cdot(75\cdot V - 20\cdot A\cdot\Omega)}{11\cdot\Omega} = 15\,A$$

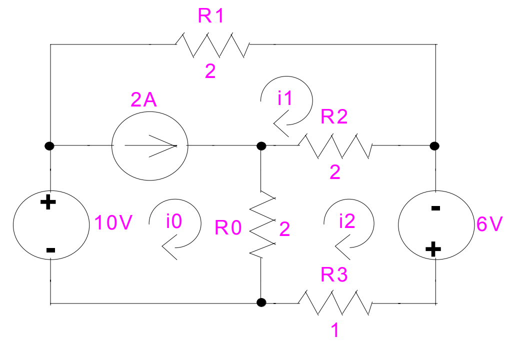
or

$$cm := \begin{pmatrix} 2\Omega + 5\Omega & -5\Omega \\ 2\Omega & -3\Omega \end{pmatrix} \qquad v := \begin{pmatrix} 75V - 10A\cdot 2\Omega \\ 0 \end{pmatrix}$$

$$cm^{-1}v = \begin{pmatrix} 15 \\ 10 \end{pmatrix} A \qquad i_a := 15A$$

AP 4.12 Use the mesh-current method to find the power dissipated in the 1 Ω resistor in the circuit shown.

$$r := \begin{pmatrix} 2 \\ 2 \\ 2 \\ 1 \end{pmatrix} \Omega$$

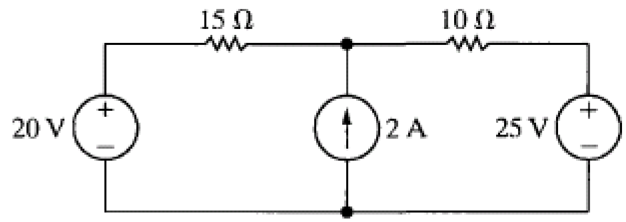


$$cm := \begin{pmatrix} r_0 & r_1 + r_2 & -r_0 - r_2 \\ -r_0 & -r_2 & r_0 + r_2 + r_3 \\ 1\Omega & -1\Omega & 0 \end{pmatrix} \qquad v := \begin{pmatrix} 10 \\ 6 \\ 2 \end{pmatrix} V$$

notes: first line is super mesh of i_0 and i_1.
second line is i_2 mesh.
third line includes current source.

$$i := cm^{-1} v = \begin{pmatrix} 7 \\ 5 \\ 6 \end{pmatrix} A \qquad P := (i_2)^2 \cdot 1\Omega = 36\,W$$

AP 4.13 Find the power delivered by the 2 A current source in the circuit shown.



$$vn := \begin{pmatrix} 1 & -1 \\ 15 & 10 \end{pmatrix} \Omega \qquad v := \begin{pmatrix} -2 \\ 20 - 25 \end{pmatrix} V$$

$$vn^{-1} v = \begin{pmatrix} -1 \\ 1 \end{pmatrix} A \qquad P := 35V \cdot 2A = 70\,W$$

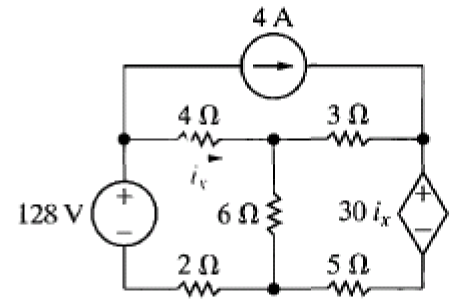
in this case the currrent mesh will result in the same equations.

$$cm := \begin{pmatrix} 15 & 10 \\ 1 & -1 \end{pmatrix} \quad v := \begin{pmatrix} 20 - 25 \\ -2 \end{pmatrix}$$

AP 4.14 Find the power delivered by the 4 A current source in the circuit shown.

with 5 resistors and only three mesh choose current mesh.

$$r := \begin{pmatrix} 2 \\ 6 \\ 4 \\ 3 \\ 5 \end{pmatrix} \Omega$$



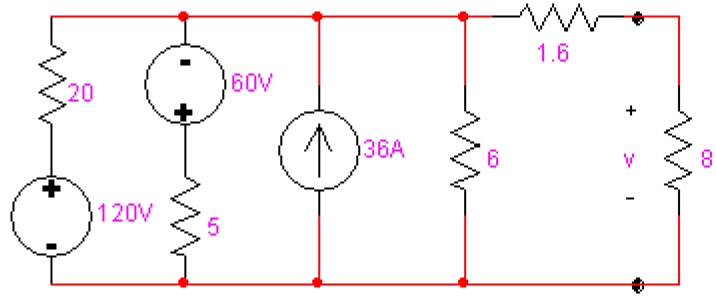
$$cm := \begin{pmatrix} r_0 + r_1 + r_2 & -r_2 & -r_1 \\ 0 & 1\Omega & 0 \\ -r_1 + 30\Omega & -r_3 - 30\Omega & r_1 + r_3 + r_4 \end{pmatrix} \qquad v := \begin{pmatrix} 128 \\ 4 \\ 0 \end{pmatrix} V$$

$$i := cm^{-1} v \qquad i = \begin{pmatrix} 9 \\ 4 \\ -6 \end{pmatrix} A \qquad \left[\left(i_0 - i_1\right) \cdot r_2 + \left(i_2 - i_1\right) r_3\right] \cdot 4A = -40\,W$$

or 40 watts delivered.

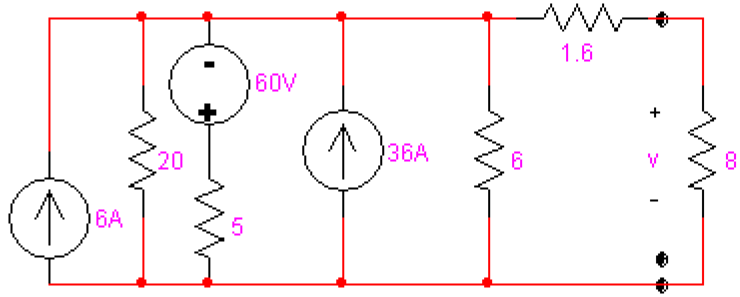
AP 4.15 a) Use a series of source transformations to find the voltage v in the circuit shown.
b) How much power does the 120 V source deliver to the circuit?

$$\|(a,b) := \frac{a \cdot b}{a+b}$$



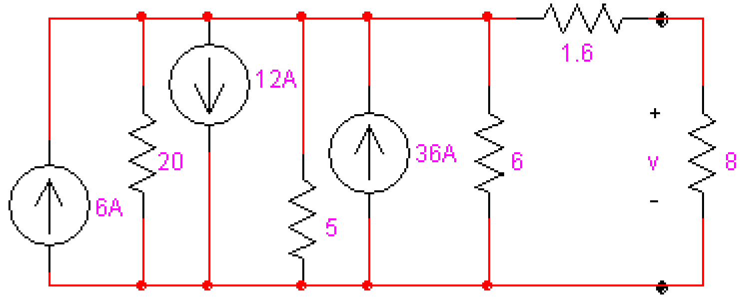
First transform the 120V 20Ω pair to Norton.

$$\frac{120V}{20\Omega} = 6\,A$$



Transform the 60V 5Ω pair to Norton

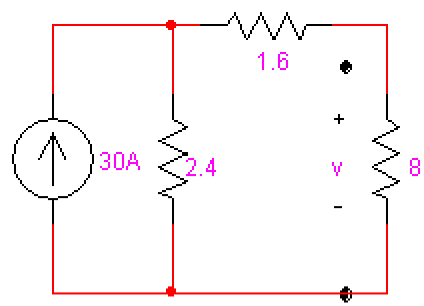
$$\frac{60V}{5\Omega} = 12\,A$$



Combine the parallel Norton sources:

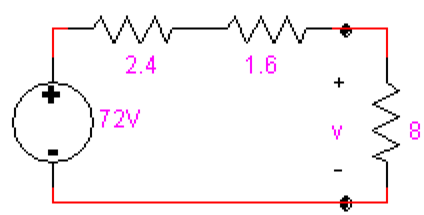
$$(20\Omega) \,\|\, (5\Omega) \,\|\, (6\Omega) = 2.4\,\Omega$$

$$6A - 12A + 36A = 30\,A$$



Transform the Norton to a Thevenin

$$30A \cdot 2.4\Omega = 72\,V$$

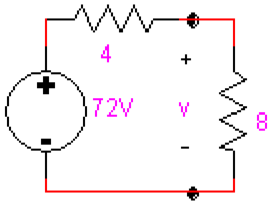


Combine the series resistors

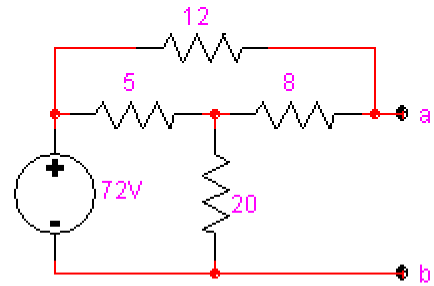
$$2.4\Omega + 1.6\Omega = 4\,\Omega$$

Use voltage divider to find answer

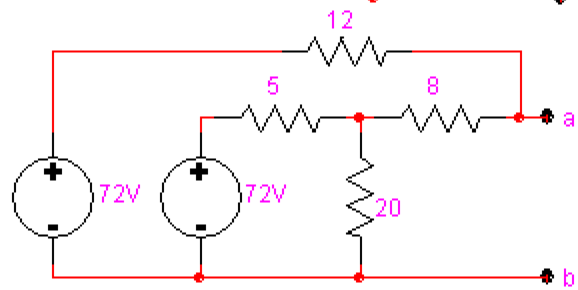
$$72\text{V}\cdot\frac{8\Omega}{8\Omega + 4\Omega} = 48\,\text{V}$$



AP 4.16 Find the Thevenin equivalent circuit with respect to the terminals a,b for the circuit shown.

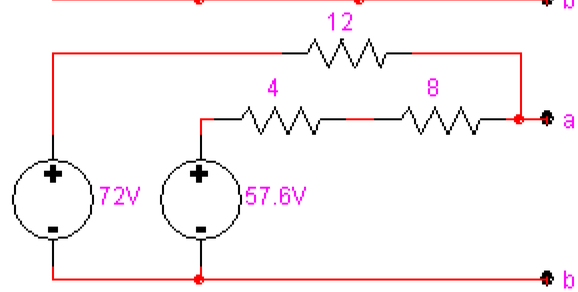


first separate the voltage source into two copies:

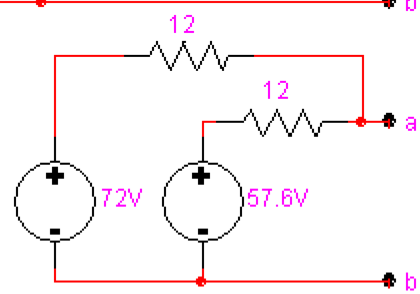


$$Vth_1 := \frac{20\Omega}{20\Omega + 5\Omega} \cdot 72V = 57.6\,V$$

$$Rth_1 := (20\Omega) \parallel (5\Omega) = 4\,\Omega$$

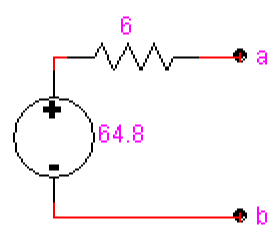


$$Rth_2 := 8\Omega + Rth_1 = 12\,\Omega$$

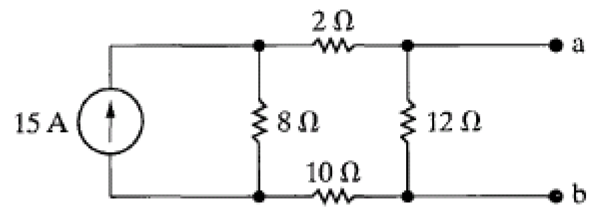


$$Rth_3 := (12\Omega) \parallel (12\Omega) = 6\,\Omega$$

$$Vth_3 := \frac{72V - Vth_1}{12\Omega + 12\Omega} \cdot 12\Omega + Vth_1 = 64.8\,V$$



AP 4.17 Find the Norton equivalent circuit with respect to the terminals a,b for the circuit shown.



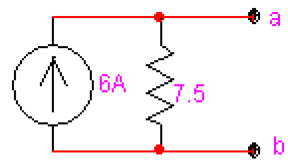
$$V_{oc} := \frac{8\Omega}{24\Omega + 8\Omega} \cdot 15A \cdot 12\Omega = 45\,V$$

$$R_{ab} := (12\Omega) \parallel [(2 + 8 + 10)\Omega] = 7.5\,\Omega \qquad I_N := \frac{V_{oc}}{R_{ab}} = 6\,A$$

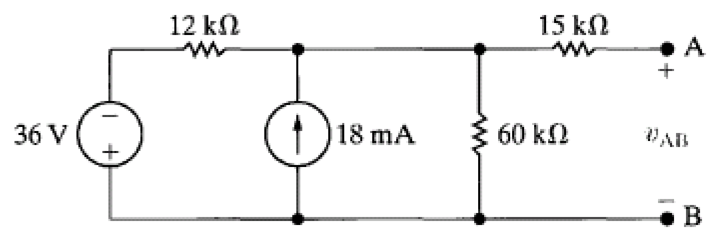
Or

$$I_{sc} := \frac{8\Omega}{(8 + 2 + 10)\Omega} \cdot 15A = 6\,A$$

$$R_N := \frac{V_{oc}}{I_{sc}} = 7.5\,\Omega$$



AP 4.18 A voltmeter with an internal resistance of 100 kΩ is used to measure the voltage v_{AB} in the circuit shown. What is the voltmeter reading?



$R_{meter} := 100k\Omega$

Norton the 36V,12kΩ to

$I_{N1} := \frac{36V}{12k\Omega} = 3 \cdot mA \quad \text{(down)}$

$R_n := (12k\Omega) \parallel (60k\Omega) = 10 \cdot k\Omega$

Thevenin the 15mA, 10kΩ

$Vth := 15mA \cdot 10k\Omega = 150\ V \qquad Rth := R_n = 10 \cdot k\Omega$

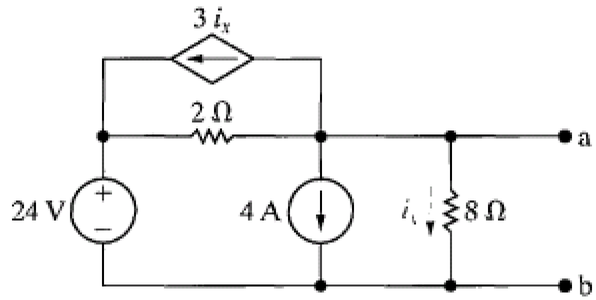
Add the series 15k

$Rth := Rth + 15k\Omega = 25 \cdot k\Omega$

The meter reading will be given by

$v_{meter} := Vth \cdot \frac{R_{meter}}{R_{meter} + Rth} = 120\ V$ (Rather sad considering the open circuit voltage is 150V.)

AP 4.19 Find the Thevenin equivalent circuit with respect to the terminals a,b for the circuit shown.



$$cm := \begin{pmatrix} 2 & 3\cdot 2 + 8 \\ 1 & -1 \end{pmatrix}\Omega \qquad v := \begin{pmatrix} 24 \\ 4 \end{pmatrix}V$$

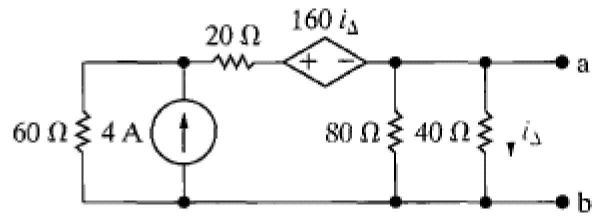
$$i := cm^{-1}v = \begin{pmatrix} 5 \\ 1 \end{pmatrix}A$$

$$V_{oc} := 8\Omega\cdot i_1 = 8\ V$$

$$I_{sc} := \frac{24V}{2\Omega} - 4A = 8\ A$$

$$V_{th} := V_{oc} = 8\ V \qquad R_{th} := \frac{V_{th}}{I_{sc}} = 1\ \Omega$$

AP 4.20 Find the Thevenin equivalent circuit with respect to the terminals a,b for the circuit shown. (Hint: Define the voltage at the leftmost node as v, and write two nodal equations with VTh as the right node voltage.)



Thevenize the 4A 60Ω to

$i_\Delta := i_\Delta \qquad V_{oc} := V_{oc}$

$$V_{th1} := 4A \cdot 60\Omega = 240\,V$$

$$R_{th1} := 60\Omega + 20\Omega = 80\,\Omega$$

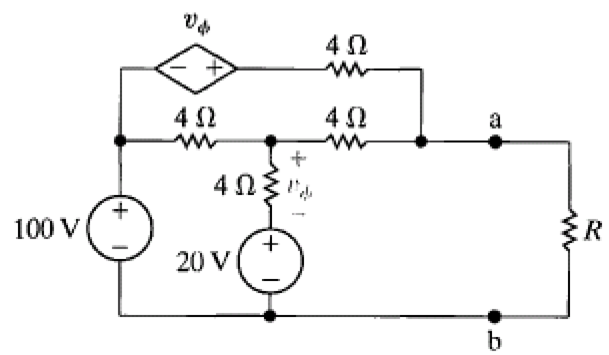
$$V_{oc} = \frac{40 \parallel 80}{(40 \parallel 80) + 80} \cdot \left(V_{th1} - 160\Omega \cdot \frac{V_{oc}}{40\Omega}\right) \text{ solve}, V_{oc} \rightarrow 30 \cdot V \qquad V_{th} := 30V$$

during a short circuit condition i_Δ be zero so the circuit will be simplified:

$$I_{sc} := \frac{60}{20 + 60} \cdot 4A = 3\,A$$

$$R_{th} := \frac{V_{th}}{I_{sc}} = 10\,\Omega$$

AP 4.21 a) Find the value of R that enables the circuit shown to deliver maximum power to the terminals a,b.
b) Find the maximum power delivered to R.



open circuit analysis:

$$cm := \begin{pmatrix} 4+4 & -4 \\ -4-4 & 4+4+4 \end{pmatrix} \cdot \Omega \quad v := \begin{pmatrix} 100-20 \\ 0 \end{pmatrix} V$$

$$i := cm^{-1} v = \begin{pmatrix} 15 \\ 10 \end{pmatrix} A$$

$$V_{oc} := 20V + i_0 \cdot 4\Omega + i_1 \cdot 4\Omega = 120\ V$$

Now analyze with a and b shorted:

$$cm := \begin{pmatrix} 4+4 & -4 & -4 \\ -4-4 & 4+4+4 & -4+4 \\ -4 & -4 & 4+4 \end{pmatrix} \Omega \quad v := \begin{pmatrix} 100-20 \\ 0 \\ 20 \end{pmatrix} V \quad i := cm^{-1} v = \begin{pmatrix} 45 \\ 30 \\ 40 \end{pmatrix} A \quad R_{th} := \frac{V_{oc}}{i_2} = 3\ \Omega$$

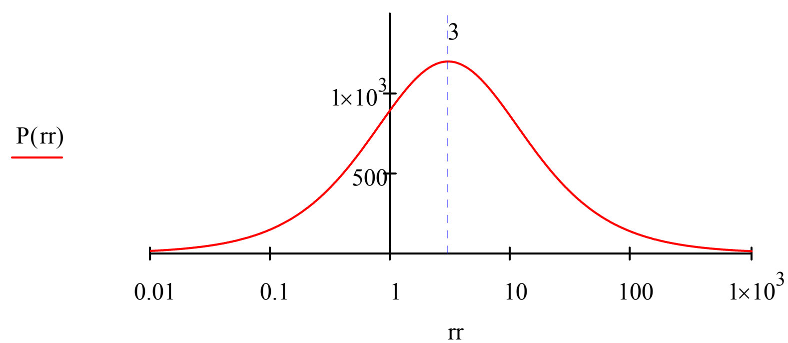
a) maximum power will be delivered to the load when R=3Ω.

b) Maximum Power delivered is: $P := \frac{V_{oc}}{2R_{th}} \cdot \frac{V_{oc}}{2} = 1.2 \cdot kW$

max power

$$V_1 := 120V \quad R_1 := 3\Omega \qquad r := r$$

$$P(r) := \frac{r}{r+R_1} \cdot V_1 \cdot \frac{1}{r+R_1} \cdot V_1 \qquad P(1\Omega) = 900\ W \qquad \frac{d}{dr} P(r) = 0 \text{ solve}, r \rightarrow 3 \cdot \Omega$$



AP 4.22 Assume that the circuit in Assessment Problem 4.21 is delivering maximum power to the load resistor R.
a) How much power is the 100 V source delivering to the network?
b) Repeat (a) for the dependent voltage source.
c) What percentage of the total power generated by these two sources is delivered to the load resistor R?

$$cm := \begin{pmatrix} 4+4 & -4 & -4 \\ -4-4 & 4+4+4 & -4+4 \\ -4 & -4 & 4+4+3 \end{pmatrix} \Omega \qquad v := \begin{pmatrix} 100-20 \\ 0 \\ 20 \end{pmatrix} V \qquad i := cm^{-1} v = \begin{pmatrix} 30 \\ 20 \\ 20 \end{pmatrix} A \qquad \left(i_2\right)^2 \cdot 3\Omega = 1.2 \cdot kW$$

a) $P_{100V} := i_0 \cdot 100V = 3 \cdot kW$

b) $P_{var} := \left(i_0 - i_2\right) \cdot 4\Omega \cdot i_1 = 800\ W$

$P_{20V} := \left(i_2 - i_0\right) \cdot 20V = -200\ W$ absorbed!

c) $\frac{1200}{3800} = 31.58 \cdot \%$