2. In a study of bowhead whales the time for a sample of whales to swim a distance of 1km was recorded. The data is contained in the file *whale.txt*. The distribution of the times t_i is to be fit using the Gamma distribution with density

$$f(t) = \frac{1}{\Gamma(\alpha)\beta^{\alpha}} t^{\alpha-1} e^{-t/\beta}$$

for $t_i > 0$, and where $\alpha > 0$ and $\beta > 0$ are parameters to be estimated. The data is contained in the file *whale.txt*. Use the **R** package *fitdistrplus* to fit the following models to the whale time data.

(a) Make a histogram of the values of t_i. Does the Gamma distribution appear to be a good model to fit to these data?

(b) Fit the Gamma distribution to the data using the *fitdist()* function from the **R** package *fitdistrplus*. Make a table to summarize the results of the fit.
   i. What are the maximum likelihood estimates of α and β?
   ii. What is the asymptotic standard error of the maximum likelihood estimate of α and β?
   iii. Calculate a asymptotic 95% confidence interval for α and β.
   iv. What is the value of the log likelihood function at the maximum likelihood estimates of α and β? Plot the log-likelihood function.
   v. What is the value Akaike Information Criterion (AIC) for the fitted model?

(c) Suppose the parameter $\alpha = 1$, what model is the model? Fit the model to the data. Make a table to summarize the results of the fit.
   i. What are the maximum likelihood estimates of β?
   ii. What is the asymptotic standard error of the maximum likelihood estimate of β?
   iii. Calculate a asymptotic 95% confidence intervals for β.
   iv. What is the value of the log likelihood function at the maximum likelihood estimates of β? Plot the log-likelihood function.
   v. What is the value Akaike Information Criterion (AIC) for the fitted model?

(d) Compare the fitted models using the *denscomp()* function from the **R** package *fitdistrplus*. Which model is the better model? Justify your answer.

(e) Using the asymptotic 95% confidence interval for parameter α in the Gamma model, test the null hyptothesis that $\alpha = 1$ at the 5% level of significance. What is your conclusion?