Exercises

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Maximum Likelihood Estimation

1. Find the MLE of θ based on a random sample $X_1, X_2, ..., X_n$ from each of the following p.d.f.'s.

   (a)
   $$f(x|\theta) = \theta x^{\theta-1}$$

   where $0 < x < 1$, $0 < \theta$, and 0 otherwise.

   (b)
   $$f(x|\theta) = (\theta+1)x^{-\theta-2}$$

   where $1 < x$, $0 < \theta$, and 0 otherwise.

   (c)
   $$f(x|\theta) = \theta^2 x e^{-\theta x}$$

   where $0 < x$, $0 < \theta$, and 0 otherwise.

   (d)
   $$f(x|\theta) = \theta(1-\theta)^{x-1}$$

   for $x = 1, 2, ...$, $0 < \theta < 1$, and 0 otherwise

2. Find the asymptotic variance of the MLE in each part of question 1.

3. Consider two independent random samples $X_1, X_2, ..., X_n \sim N(\mu, \sigma_1^2)$ and $Y_1, Y_2, ..., Y_m \sim N(\mu, \sigma_2^2)$.

   (a) Using the data from the two random samples find the m.l.e. of μ, σ_1^2, and σ_2^2.

   (b) Find the asymptotic variance of μ.