Bayesian Deconvolution of Seismic Array Data Using the Gibbs Sampler

Eric A. Suess and Robert Shumway, University of California, Davis
Rong Chen, Texas A&M University
Contact: Eric A. Suess, Division of Statistics, One Shields Avenue, Davis, CA 95616
(e-mail: suess@wald.ucdavis.edu)

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Abstract

The problem of monitoring for low magnitude nuclear explosions, using seismic array data, under a Comprehensive Test Ban Treaty (CTBT) requires a capability for distinguishing nuclear explosions from other seismic events. Industrial mining explosions are one type of seismic event that needs to be ruled out when trying to detect nuclear tests. We consider a Bayesian approach to the problem of detecting ripple-fired mining explosions. Seismic array data are expressed as multidimensional time domain convolutions of an unknown pulse function, representing the ripple-fired delay pattern, with unknown signal-path effects sequences on each channel, which are assumed to follow independent AR processes. Using the Gibbs Sampler and a proposal of Cheng, Chen and Li [1] for blind deconvolution, we develop an approach to estimating the delay parameters and the unknown signal-path effects sequence at each sensor. Results for a ripple-fired mining explosion recorded at the Arctic Experimental Seismic Station (ARCESS) will be presented. Finally, the implications of our method for monitoring a nuclear test ban treaty are considered.

1 Introduction

The problem of monitoring seismic events for possible nuclear explosions is an important one that has been studied extensively and is still open to further development. Much work has been done on this problem due to the many treaties that have been signed in the past between the U.S., the former U.S.S.R., and other nuclear powers related to testing issues. The focus in the past has been on distinguishing possible nuclear explosions from earthquakes. Currently, because the above treaties have put limitations on the permissible sizes of the nuclear explosions for testing other smaller seismic events such as mining explosions have become of interest in the discrimination problem. The work we are proceeding with here is related to distinguishing low level nuclear explosions from ripple-fired mining explosions that are on the same seismic level. This technique of quarry mining involves the use of multiple rows of explosives that are detonated with approximately equal time delays between the rows.

The objective of this paper is to develop an estimation procedure that estimates delay times in seismic events. The delay structure of a single seismic event is characterized by the size (or amplitude) of the delayed explosions and the delay times between the single explosions. If a delay structure can be estimated, this will be useful in discriminating ripple-fired mining explosions from other, possibly nuclear, seismic events. In addressing this problem, we have developed a model for seismic recordings collected by an array of sensors for ripple-fired events and a deconvolution method based on work by Rong Chen, in the univariate case, and work by Cheng, Chen, and Li [1] that is used to estimate parameters in our model related to the delay time structure.

Our method works by separating, or deconvolving a pulse sequence from the underlying signal-path effects sequences. The pulse sequence is used to model the detonation design of a mining explosion. The underlying signal-path effects sequences are used to model the common underlying signal sent by each detonation in a ripple-fired event and the effects of the path on the signal as it traces to the sensors. We approach the problem from a Bayesian perspective. The Gibbs sampler is implemented to produce posterior estimates of the parameters in our model, using the data and available prior information. From the estimated pulse sequence we produce estimates of the delay time structure of ripple-fired events.

2 Formulation of the Problem

We model the k^{th} seismic trace $y_k(t)$, $t = 1, ..., n$, in an array of q-sensors, that is suspected to have been produced by a ripple-fired mining explosion, using the following multivariate convolution model:

$$y_k(t) = \sum_{j=0}^{m} a_j s_k(t-j) + \varepsilon_k(t). \qquad (1)$$

We refer to the vector of parameters $\mathbf{a} = [a_1, ..., a_m]'$ as the relative pulse sequence. This set of parameters is assumed to reflect the delay pattern used in the design of the ripple-fired event. We define the k^{th} underlying signal-path effects sequence, $s_k(t)$, $t = 1, ..., n$, as the combination of the "signal" produced by each sub-explosion in a ripple-fired event along with the "path effects" that are due to the random imperfections in the material the signal travels through on the k^{th} channel to the k^{th} seismometer. We assume that the pulse sequence $\mathbf{a}$ is independent of the underlying signal-path effects sequence $s_k(t)$, $t = 1, ..., n$, on each channel k, $k = 1, .., q$. Assuming that the first explosion in the ripple-fired event is used as the reference signal, for which we fix a_0 to be 1, and that the remaining a_j are calculated relative to a_0, the model can be written in a more insightful form as

$$y_k(t) = s_k(t) + \sum_{j=1}^{m} a_j s_k(t-j) + \varepsilon_k(t). \qquad (2)$$

This model assumes that each delayed explosion in a ripple-fired event sends a signal having the same form as the first. The only difference is that its amplitudes a_j may vary.

Further modeling assumptions on the pulse sequence $\mathbf{a}$, and the underlying signal-path effects sequences $s_k(t)$, $k = 1, .., q$, are needed to insure that the parameters in the convolution model are identifiable. To model the pulse sequence we use a variant of the Bernoulli-Gaussian model commonly used to model reflection seismology data. A discussion of the model can be found in Mendel [4]. For the pulse sequence $\mathbf{a}$, we fix $a_0 = 1$ and model the relative amplitudes as follows: We define X_j, $j = 1, ..., m$, to be a sequence of independent Bernoulli random variables with $p(X_j = 1) = 1 - \eta$ and $p(X_j = 0) = \eta$. The nonzero values of X_j index the possible detonation times in the ripple-fired event of duration m, measured in points per second. The parameter η is the probability of observing a zero value in the pulse sequence. The parameter m is the length of the vector $\mathbf{a}$, m is assumed to be known. Next, we define the distribution of each a_j, conditional on X_j, $j = 1, ..., m$. In ripple-fired events the amplitudes will be positive because mining explosions are usually performed at shallow depths. We define a_j, $j = 1, ..., m$, conditional on X_j, $j = 1, ..., m$, as $p(a_j | X_j = 0) = 1$ if $a_j = 0$ and 0 otherwise. And

$$\begin{aligned} p(a_j | X_j = 1) &= c^{-1}(\mu_\alpha, \sigma_\alpha^2) \frac{1}{\sqrt{2\pi}\sigma_\alpha} \\ &\quad \times \exp\left\{ -\frac{(a_j - \mu_\alpha)^2}{\sigma_\alpha^2} \right\} I(a_j > 0), \end{aligned} \qquad (3)$$

where

$$c^{-1}(\mu_\alpha, \sigma_\alpha^2) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left\{ -\frac{(x - \mu_\alpha)^2}{\sigma_\alpha^2} \right\} dx.$$

and $I(a_j > 0) = 0$ if $a_j > 0$ and 0 otherwise. The truncated normal distribution is used because the a_j, the relative amplitude of the delayed detonation at time j, is assumed to be positive and random. Truncating at zero restricts the values of a_j to positive values.

It is reasonable to use the truncated normal distribution to model the variation in the size of a single detonation since the distribution of the size of a single explosion should be centered at a mean value μ_α with a variance σ_α^2. Alternatively, we write the density of the independent a_j, $j = 1, ..., m$, as

$$p(a_j | \eta) = \eta I(a_j = 0) + (1 - \eta) p(a_j | X_j = 1) I(a_j > 0), \qquad (4)$$

which is a mixture of a Bernoulli distribution and a truncated Gaussian distribution. The mean μ_α is used to model the average size of the delayed explosions in ripple-fired events in the monitoring region and the variance σ_α^2 is used to model the variance of such explosions. So the a_j are independent and identically distributed with a density defined in (4). Note that for the Bernoulli sequence X_j, $j = 1, ..., m$, will have n_α 0 values and $m - n_\alpha$ unit values, which implies that the pulse sequence has $m - n_\alpha$ nonzero values or pulses. Note that n_α is random. Define J_α to be the index set that includes the values of j for nonzero values of X_j or thereby the nonzero values of a_j. Hence, we define the density of $\mathbf{a} = [a_1, ..., a_m]'$ as

$$\begin{aligned} p(\mathbf{a} | \eta) &= \eta^{n_\alpha} (1 - \eta)^{m - n_\alpha} c^{-n_\alpha}(\mu_\alpha, \sigma_\alpha^2) \left[\frac{1}{\sqrt{2\pi}\sigma_\alpha} \right]^{n_\alpha} \\ &\quad \times \exp\left\{ -\frac{1}{2\sigma_\alpha^2} \sum_{J_\alpha} (a_j - \mu_\alpha)^2 \right\}. \end{aligned} \qquad (5)$$

This modeling assumption is based on the Bernoulli-Gaussian model used by Cheng, Chen, and Li [1].

To model the underlying signal-path effects sequence $s_k(t)$, in the k^{th} seismic trace $y_k(t)$, $t = 1, ..., n$, we use a low-order autoregressive, $AR(p)$, model

$$\phi_0 s_k(t) + \phi_1 s_k(t-1) + ... \\ + \phi_p s_k(t-p) = e_k(t), \qquad (6)$$

where we fix $\phi_0 = 1$ and let $e_k(t)$ be independent and identically distributed, over t and k, normal with mean 0 and variance σ^2. The use of this model is justified by Dargahi-Noubary [2] and Tjøstheim [8] who gave theoretical and empirical arguments for modeling seismic waveforms using low-order autoregressive models. The precision of $s_k(t)$ is defined as $\tau = 1/\sigma^2$, which is used later in our application of the Gibbs sampler. We use the same set of autoregressive coefficients, $\boldsymbol{\phi} = [\phi_1, ..., \phi_p]'$, to model the k^{th} underlying signal-path effects sequence $s_k(t)$, since the same event is assumed to generate the trace $y_k(t)$ recorded at the k^{th} seismometer. Hence, we believe the same autoregressive model will be applicable for each underlying signal-path effect sequence $s_k(t)$, $k = 1, ..., q$, $t = 1, ..., n$, although the functions generated will be different.

Finally, we assume that the additive observation noise term $\varepsilon_k(t)$ in (2) is distributed independent and identically, over t and k, normal with mean 0 and variance $c\sigma^2$, where $c > 0$ is fixed. The parameter c can be thought of as the inverse of the signal-to-noise ratio, defined as $SNR = \sigma_e^2/\sigma_\varepsilon^2$, where σ_e^2 is the variance of the underlying signal-path effects sequence on each channel and σ_ε^2 is the variance of the noise term. Recall that $\sigma_e^2 = \sigma^2$ and $\sigma_\varepsilon^2 = c\sigma^2$, which implies that $SNR = 1/c$. The use of the fixed parameter c alleviates a scaling problem that exists if two separate variance terms are included in our model.

3 The Bayesian Approach

In this paper the deconvolution of seismic array data for common delay patterns is achieved using Bayesian methods. Specifically in the model under consideration in (1), the parameter set is

$$\boldsymbol{\Theta} \;\; = \;\; \{\eta, \mathbf{a}, \boldsymbol{\phi}, \tau, \mathbf{S}\}. \qquad (7)$$

The set $\boldsymbol{\Theta}$ contains $1+m+p+1+nq$ elements, where we define $\mathbf{a} = [a_1, a_2, ..., a_m]'$, $\boldsymbol{\phi} = [\phi_1, \phi_2, ..., \phi_p]'$, and $\mathbf{S} = [\mathbf{s}_1, \mathbf{s}_2, ..., \mathbf{s}_q]'$, such that each row of $\mathbf{S}$ is $\mathbf{s}_k = [s_k(1), s_k(2), ..., s_k(n)]'$. The given data consists of the collection of q seismic traces recorded for a duration of n points. Here we define $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_q]'$, such that each row of $\mathbf{Y}$ is $\mathbf{y}_k = [y_k(1), y_k(2), ..., y_k(n)]'$. Hence in this problem only qn data points are available to estimate all of the parameters in $\boldsymbol{\Theta}$.

To develop a Bayesian deconvolution method we follow the Bayesian approach to statistical data analysis. We develop a model and likelihood $p(\mathbf{Y}|\boldsymbol{\Theta})$. Prior distributions for the parameters are chosen. The overall prior is $p(\boldsymbol{\Theta})$. Lastly, the posterior distributions of the parameters in $\boldsymbol{\Theta}$, given the data $\mathbf{Y}$, is calculated. By using Bayes' Rule the posterior distributions can be calculated as $p(\boldsymbol{\Theta}|\mathbf{Y}) \propto p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})$. Then, to draw inferences about the unknown parameters in our model, we calculate the marginal posteriors. For point estimates we calculate the posterior means.

In our deconvolution problem, $\boldsymbol{\Theta}$ is a vector of many parameters, and due to our modeling assumptions the marginal posterior distributions are difficult to calculate. However, the conditional marginal distributions are available. From the latter we can implement the Gibbs sampler, a Markov Chain Monte Carlo technique that simulates "random samples" from the conditional marginal distributions, which can be used to calculate posterior estimates of the parameters in our model. Point estimates are calculated by taking the means of the Markov chains samples after a sufficient "burn in" period. See Gelfand and Smith [3] and Tanner [7] for descriptions of the Gibbs sampler.

3.1 Prior Distributions

To perform our Bayesian analysis we choose prior distributions for each unknown parameter in the parameter set $\boldsymbol{\Theta}$. We specify priors on η, $\boldsymbol{\phi}$, and τ since they are unknown. We have modeling assumptions that determine the distributions of the random parameters $\mathbf{a}$ and $\mathbf{S}$. The probability η of seeing a zero at any point in the pulse sequence a_j, $j = 1, ..., m$, is assumed to have a beta prior, with predetermined hyperparameters $\beta_1 > 0$ and $\beta_2 > 0$. For the $AR(p)$ coefficients $\boldsymbol{\phi} = [\phi_1, \phi_2, ..., \phi_p]'$, we assume a p-variate normal prior distribution where $\boldsymbol{\phi}_0$ and Σ_0 are predetermined hyperparameters. To specify the prior for the common precision of the underlying signal-path effects, for each channel k, we assume that τ has a gamma prior where $\gamma_1 > 0$ and $\gamma_2 > 0$ are predetermined hyperparameters.

3.2 Overall Prior, Likelihood, and Conditional Posterior Distributions

The overall prior distribution on the parameter set $\boldsymbol{\Theta}$ is defined using independence assumptions and is

$$p(\boldsymbol{\Theta}) = p(\eta)p(\boldsymbol{\phi})p(\tau)\prod_{j=1}^{m} p(a_j|\eta)\prod_{k=1}^{q} p(\mathbf{s}_k|\boldsymbol{\phi},\tau). \quad (8)$$

We specify the conditional likelihood and use it as an approximation of the full likelihood. Due to the dependence of $y_k(t)$ on the past m values of $s_k(t)$, which can be seen in (1), and the dependence of $s_k(t)$ on its past p values, which can be seen in (6), we define the likelihood conditional on $y_k(1), ..., y_k(l)$, $k = 1, ..., q$, where $l = m + p$. We now redefine the data matrix as $\mathbf{Y} = [\mathbf{y}_1, \mathbf{y}_2, ..., \mathbf{y}_q]$, where each $\mathbf{y}_k$, $k = 1, ..., q$, starts at $l + 1 = (m + p) + 1$, so $\mathbf{y}_k = [y_k(l + 1), ..., y_k(n)]'$. Using the model assumption that the noise, $\varepsilon_k(t)$, is independent and identically distributed normal with mean 0 and variance σ^2, the conditional likelihood is

$$p(\mathbf{Y}|\boldsymbol{\Theta}) = (2\pi)^{-q(n-l)/2}\left(\frac{\tau}{c}\right)^{q(n-l)/2} \times \exp\left\{-\frac{\tau}{2c}\sum_{k=1}^{q}\sum_{t=l+1}^{n}\varepsilon_k^2(t)\right\} \quad (9)$$

where $\varepsilon_k(t) = y_k(t) - \sum_{j=0}^{m} a_j s_k(t-j)$. The joint density of the unknown parameters $\boldsymbol{\Theta}$ and the data $\mathbf{Y}$ is $p(\mathbf{Y},\boldsymbol{\Theta}) = p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})$, where the prior is given in (8) and the likelihood is given in (9). The joint posterior of the unknown parameters, given the data, can be expressed using Bayes' Rule as proportional to the likelihood times the prior, here $p(\boldsymbol{\Theta}|\mathbf{Y}) \propto p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})$. For our deconvolution, calculations of $p(\eta|\mathbf{Y})$, $p(\mathbf{a}|\mathbf{Y})$, $p(\boldsymbol{\phi}|\mathbf{Y})$, $p(\tau|\mathbf{Y})$, and $p(\mathbf{S}|\mathbf{Y})$ are not easily calculated analytically. So we now proceed to develop the conditional marginal posterior distributions of the parameters in our model, which are used in the Gibbs sampler. It is through the Gibbs sampler that we produce estimates of the parameters in our model.

We proceed to develop the conditional marginal posterior distributions of the sets of parameters in our model. The following conditional marginal posteriors distributions are calculated: $p(\eta|\mathbf{Y}, rest)$, $p(\mathbf{a}|\mathbf{Y}, rest)$, $p(\boldsymbol{\phi}|\mathbf{Y}, rest)$, $p(\tau|\mathbf{Y}, rest)$, and $p(\mathbf{S}|\mathbf{Y}, rest)$, where $rest$ refers to the remaining parameters in $\boldsymbol{\Theta}$ for a specific parameter or set of parameters of interest.

1. The conditional posterior of η is

$$\eta|\mathbf{Y}, rest \sim beta(\beta_1^*, \beta_2^*), \quad (10)$$

with parameters $\beta_1^* = n_\alpha + \beta_1$ and $\beta_2^* = m - n_\alpha + \beta_2$.

2. The conditional posterior of a_j, $j = 1, ..., m$ is

$$p(a_j|\mathbf{Y}, rest) \propto \eta_j I(a_j = 0) + (1-\eta_j)c^{-1}(\mu_{a_j}\sigma_{a_j}^2)\frac{1}{\sqrt{2\pi}\sigma_{a_j}} \times \exp\left\{-\frac{1}{2}\frac{(a_j-\mu_{a_j})^2}{\sigma_{a_j}^2}\right\} \times I(a_j > 0), \quad (11)$$

where $\eta_j = \eta/[\eta + (1-\eta)d_j]$,

$$d_j = \left[\frac{c(\mu_{a_j}, \sigma_{a_j}^2)}{c(\mu_\alpha, \sigma_\alpha^2)}\right]\left(\frac{\sigma_{a_j}}{\sigma_\alpha}\right)\exp\left\{\frac{1}{2}\left[\frac{\mu_{a_j}^2}{\sigma_{a_j}^2} - \frac{\mu_\alpha^2}{\sigma_\alpha^2}\right]\right\},$$

$$\mu_{a_j} = \sigma_{a_j}^2\left[\frac{\mu_\alpha}{\sigma_\alpha^2} + \frac{\tau}{c}\sum_{k=1}^{q}\sum_{t=l+1}^{n}\varepsilon_k^*(t[-j])s_k(t-j)\right]$$

where $\varepsilon_k^*(t[-j]) = y_k(t) - \sum_{i=0,i\neq j}^{m} a_i s_k(t-i)$, and

$$\sigma_{a_j}^2 = \left[\frac{1}{\sigma_\alpha^2} + \frac{\tau}{c}\sum_{k=1}^{q}\sum_{t=l+1}^{n}s_k^2(t-j),\right]^{-1}.$$

3. The conditional posterior of $\boldsymbol{\phi}$ is

$$\boldsymbol{\phi}|\mathbf{Y}, rest \sim N_p(\boldsymbol{\phi}_*, \Sigma_*), \quad (12)$$

where $\tilde{\mathbf{s}}_k(t) = [s_k(t-1), ..., s_k(t-p)]'$,

$$\boldsymbol{\phi}_* = \Sigma_*\left[-\tau\sum_{k=1}^{q}\sum_{t=l+1}^{n}s_k(t)\tilde{\mathbf{s}}_k(t) + \Sigma_0^{-1}\boldsymbol{\phi}_0\right],$$

and

$$\Sigma_*^{-1} = \tau\sum_{k=1}^{q}\sum_{t=l+1}^{n}\tilde{\mathbf{s}}_k(t)\tilde{\mathbf{s}}_k'(t) + \Sigma_0^{-1}.$$

4. The conditional posterior of τ is

$$\tau|\mathbf{Y}, rest \sim gamma(\gamma_1^*, \gamma_2^*), \quad (13)$$

where $\gamma_1^* = q(2n - l - p)/2 + \gamma_1$ and

$$\gamma_2^* = \frac{1}{2c}\sum_{k=1}^{q}\sum_{t=l+1}^{n}\varepsilon_k^2(t) + \frac{1}{2}\sum_{k=1}^{q}\sum_{t=p+1}^{n}e_k^2(t) + \gamma_2.$$

5. For $k = 1, .., q$ and $i = (p+1), ..., n$, $s_k(i)$ has a Normal posterior distribution with

$$s_k(i)|\mathbf{Y}, rest \sim N(\mu_{s_k(i)}, \sigma_{s_k(i)}^2), \quad (14)$$

with

$$\mu_{s_k(i)} = \sigma^2_{s_k(i)} \left[\frac{\tau}{c} \sum_{t=max(i,(l+1))}^{min((i+m),n)} \varepsilon'_k(t[-i]) a_{t-i} - \tau \sum_{t=i}^{min((i+p),n)} e'_k(t[-i]) \phi_{t-i} \right]$$

where

$$\varepsilon'_k(t[-i]) = y_k(t) - \sum_{j=0, j\neq t-i}^{m} a_j s_k(t-j),$$

$$e'_k(t[-i]) = \sum_{j=0, j\neq t-i}^{p} \phi_j s_k(t-j),$$

and

$$\sigma^2_{s_k(i)} = \left[\frac{\tau}{c} \sum_{t=max(i,(l+1))}^{min((i+m),n)} a^2_{t-i} + \tau \sum_{t=i}^{min((i+p),n)} \phi^2_{t-i} \right]^{-1}.$$

6. For $k = 1, .., q$ the conditional posterior of

$$\tilde{\mathbf{s}}_k(p+1) = [s_k(p), ..., s_k(1)]'.$$

We define $\Phi_t = [\phi_t, ..., \phi_p, 0, ..., 0]'$ for $t = 1, ..., p$ and also define

$$c_k(t) = s_k(p+t) + \phi_1 s_k(p+t-1) + ... + \phi_{t-1} s_k(p+1),$$

for $t \in \{1, ..., p\}$. Therefore, we have

$$\tilde{\mathbf{s}}_k(p+1) \sim N_p(\mu_{\tilde{\mathbf{s}}_k(p+1)}, \Sigma_{\tilde{\mathbf{s}}'_k(p+1)}) \quad (15)$$

where

$$\mu_{\tilde{\mathbf{s}}_k(p+1)} = \Sigma_{\tilde{\mathbf{s}}_k(p+1)} \left[-\frac{\tau}{c} \sum_{t=1}^{p} c_k(t) \Phi_t \right]$$

and

$$\Sigma^{-1}_{\tilde{\mathbf{s}}'_k(p+1)} = \left[\frac{\tau}{c} \sum_{t=1}^{p} \Phi_t \Phi'_t + \Sigma^{-1}_{\phi} \right].$$

4 Results for Real Data

We present in this section the results produced by applying our Bayesian deconvolution technique to real data. The array data we analyze is Event 054, a mine blast recorded at the Arctic Experimental Seismic Station (ARCESS) in northern Norway, previously analyzed by Shumway, Baumgart, and Der [5]. The seismic source of this data is known to be a ripple-fired mining explosion that was recorded at a regional distance with a sampling rate of 40 points per second.

4.1 Hyperparameters and Fixed Model Parameters

In this section we first present our choice of hyperparameters for the prior distributions on the parameters in $\mathbf{\Theta}$ and our choice of the fixed model parameters. For η, the hyperparameters we use in our beta prior are $\beta_1 = 43$ and $\beta_2 = 17/3$. For the autoregressive coefficients $\boldsymbol{\phi}$ the mean value we use for our multivariate normal prior on $\boldsymbol{\phi}$ is $\boldsymbol{\phi}_0 = [-0.9, 0.7, -0.2]'$ and the covariance matrix is

$$\Sigma_0 = \begin{bmatrix} (0.45)^2 & -(0.25)^2 & (0.15)^2 \\ -(0.25)^2 & (0.30)^2 & -(0.15)^2 \\ (0.15)^2 & -(0.15)^2 & (0.15)^2 \end{bmatrix}. \quad (16)$$

The hyperparameters we choose for the gamma prior distribution of the precision of the underlying signal-path effects sequence $\mathbf{s}_k$ on each channel k are $\gamma_1 = 5$ and $\gamma_2 = 4$. The hyperparameters that determine the normal distribution used to model the independent nonzero elements of the pulse sequence a_j are $\mu_\alpha = 0.7$ and $\sigma^2_\alpha = 0.15^2$.

The value of the parameter c is chosen to reflect the signal-to-noise ratio defined as $SNR = \sigma^2_e / \sigma^2_\varepsilon$. We fix $c = 0.01$. And we fix $m = 40$.

4.2 Results for Event 054

Here we present the results produced by applying our Bayesian deconvolution program to the P_n-phase of Event 054, see Figure 1 for plots of the $q = 5$ recordings of the P_n-phase. Estimates of the model parameters $\mathbf{\Theta}$ are calculated from a typical run of our Gibbs sampling program. For the results presented here the program was run 40,000 iterations and a "burn in" of 35,000 iterations was used. The estimates were computed by calculating the means of the Markov chains after "burn in" for each parameter.

The peak values of $\mathbf{a}$ estimated from the P_n-phase occur at the times 4, 8, 12, 14, 18, 25, 28, and 35, which can be seen in Figure 2. The estimated delay structure contains multiple delays, indicating that the P_n-phase of Event 054 was generated by ripple-firing.

5 Conclusions

To summarize our work, we have developed a Bayesisan deconvolution method for seismic array data that detects a common delay pattern. This work was based on the univariate work of Rong Chen and and Cheng, Chen, and Li [1]. The priors we chose were derived from the previous work

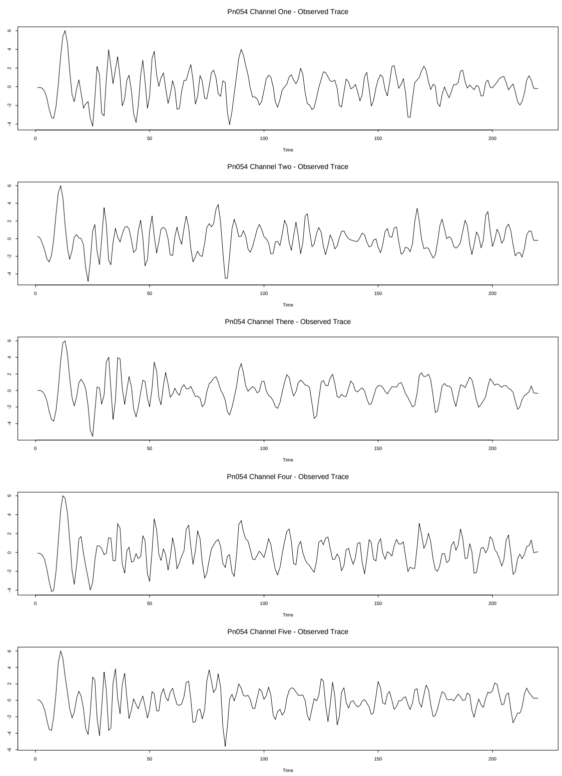

Figure 1: P_n-phase Event 054: Scaled data, $\mathbf{y}_k$, $k = 1, ..., q$, where $q = 5$ and $n = 220$.

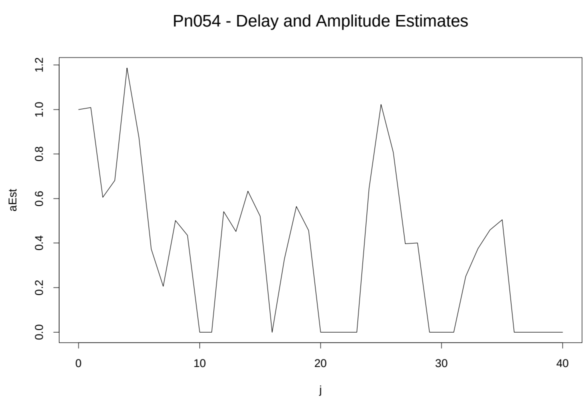

Figure 2: P_n-phase Event 054: Posterior estimates of the *pulse sequence*, $\hat{a}_j$, $j = 1, ..., m$, where $m = 40$.

of Tjøstheim [8], Smith [6], and Shumway, Baumgart, and Der [5]. We tested our program extensively on simulated data and we used the lessons learned from the simulated data analysis to implement our program on the data from Event 054 recorded at ARCESS.

We foresee two main implications of our method for monitoring seismic activity for low level nuclear explosions. First, our method can incorporate into the statistical analysis expert opinion related to the parameters in our model or research related to the seismic activity of the region being monitored. Secondly, we see the possibility of automating the analysis so a quick first analysis can be run to check seismic events for common ripple-fired mining explosions.

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