

Final

Instructions: This is an open book, open notes exam. Write your answers on your own paper. Show your work for full credit.

1. (**25 points**) Find the best linear forecast of X_{n+1} and X_{n+2} using the prediction operator P_n when X_t follows an $SARIMA(1, 0, 0)(1, 0, 0)_{12}$ model.
2. (**35 points**) **Short answer.**
 - (a) What is the main assumption made about time series data that distinguishes it from the assumptions of linear regression data?
 - (b) Define stationarity. Sketch a time series that is nonstationary and one that is stationary.
 - (c) Sketch the general form of the ACF of a nonstationary time series.
 - (d) Sketch the ACF of a stationary time series that has a seasonal correlation at lag 12.
 - (e) If X_t and Y_t are two correlated time series, what diagnostic time series function is used to detect the correlation between two such time series?
 - (f) What is the difference between the Time Domain and Frequency Domain approaches to time series analysis?
 - (g) Sketch a time series that contains two separate, clearly distinguishable, frequencies.
3. (**40 points**) This problem uses the pictures on the next few pages to develop an $SARIMA(p, d, q)(P, D, Q)_s$ model for the monthly U.S. birth data from most of this century.
 - (a) The Birth data is nonstationary so a single difference, $d = 1$ is necessary. From the spectrum of the raw data, what is the most important frequency in the data?
 - (b) Looking at the differenced data in the second picture the ACF shows a seasonal nonstationarity, this implies, $D = 1$, with $s = 12$. What does the PACF suggest that the order of P should be?
 - (c) It is assumed that $P = 2$. The following model was fit to the raw Birth data, $SARIMA(0, 1, 0)(2, 1, 0)_{12}$ and the residuals are shown the third picture. What does the ACF of the residuals suggest the order of q should be?
 - (d) It is assumed that $q = 1$. So the following model was fit to the raw Birth data, $SARIMA(0, 1, 1)(2, 1, 0)_{12}$ and the residuals are shown the fourth picture. Do the residuals seem to be white? Explain.







