Stat 6871
Spring 1999

Midterm 1 Solution

(1) If Y_t has a quarterly seasonal trend it is nonstationary. The Classical Decomposition Model is

$$Y_t = m_t + s_t + W_t$$

where $m_t = \beta_0$, $s_t = s_{t-4}$. So

$$E[Y_t] = \beta_0 + s_t,$$

which depends on time. The ∇_4 will transform the data to a stationary time series

$$\nabla_4 Y_t = \nabla_4(\beta_0 + s_t + W_t)$$

$$= \beta_0 + s_t + W_t - (\beta_0 + s_{t-4} + W_{t-4})$$

$$= s_t - s_{t-4} + W_t - W_{t-4}$$

$$= W_t - W_{t-4}$$

$$\therefore E[\nabla_4 Y_t] = 0$$

and

$$\gamma_{\nabla_4 y}(h) = E[(w_{t+h} - w_{t+h-4})(w_t - w_{t-4})]$$

$$= E[w_{t+h} w_t] - E[w_{t+h} w_{t-4}]$$

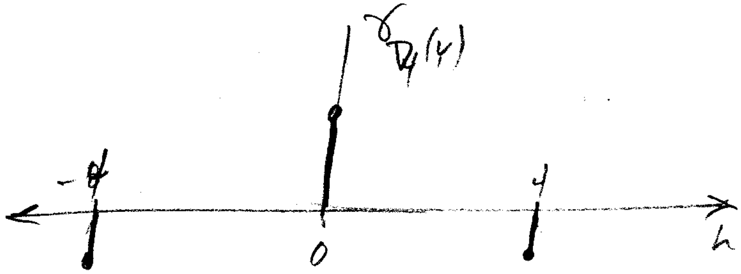
$$- E[w_{t+h-4} w_t] + E[w_{t+h-4} w_{t-4}]$$

$h=0 \quad \gamma_{\nabla_4 y}(0) = \sigma^2 + \sigma^2 = 2\sigma^2$

$|h|=1,2,3 \quad \gamma_{\nabla_4 y}(1) = 0$

$|h|=4 \quad \gamma_{\nabla_4 y}(2) = -\sigma^2$

$|h|>4 \quad \gamma_{\nabla_4 y}(h) = 0$



(2) a) $E[Y_t] = \beta_0 + \beta_1 t + \beta_2 t^2$ so X_t

is nonstationary since it depends on time.

b) $\nabla^2 Y_t = \nabla(\nabla Y_t) = \nabla(Y_t - Y_{t-1})$

$= (Y_t - Y_{t-1}) - (Y_{t-1} - Y_{t-2})$

$= Y_t - 2Y_{t-1} + Y_{t-2}$ plug in and solve.

$= Z_t - 2Z_{t-1} + Z_{t-2} + 2\beta_2$

$E[\nabla^2 Y_t] = 2\beta_2$ doesn't depend on t

$\gamma_{\nabla^2 Y_t}(h) = E[(Z_{t+h} - 2Z_{t+h-1} + Z_{t+h-2})$

$\cdot (Z_t - 2Z_{t-1} + Z_{t-2})]$

$h=0$ $\gamma_{\nabla^2 Y_t}(0) = 6\sigma^2$ doesn't depend on t

$|h| = 1$ $= -4\sigma^2$

$|h| = 2$ $= \sigma^2$

$|h| \geq 3$ 0 $\therefore$ $\nabla^2 Y_t$ is stationary

(3.) a) ARMA(2, 0)

$$(1-1.05B+0.4B^2)X_t = Z_t$$

$$\phi_1 = 1.05 \qquad \phi_2 = -.4$$

conditions $|\phi_2| = .4 < 1$,

$$\phi_2 + \phi_1 = .65 < 1, \text{ and}$$

$$\phi_2 - \phi_1 = -1.45 < 1$$ so stationary

- OR -

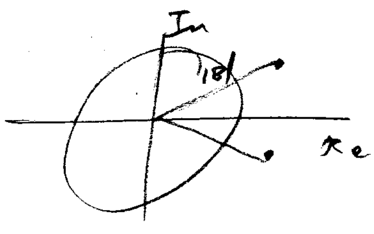
$$1 - 1.05B + .4B^2 = 0$$

roots

$$B = \frac{1.05 \pm \sqrt{1.05^2 - 4(.4)(1)}}{2(.4)}$$

$$= 1.3125 \pm i(.62180)$$

$$|B| = 2.1094 > 1$$



Since the roots lie outside the unit circle, stationary.

b) $(1-1.05)X_t = Z_t$

$|\phi_1| = 1.05 \not< 1$ not stationary

—OR—

$1-1.05B = 0 \Rightarrow B = .9524$

$\Rightarrow |B| < 1$ root insio unit circle

ARMA(1,1)

c) $(1+.8B)X_t = (1-.25B)Z_t$

$\phi_1 = -.8 \qquad \theta = -.25$

$|\phi_1| = .8 < 1$ stationary

—OR—

$1+.8B = 0 \Rightarrow B = -1.25$

$\Rightarrow |B| > 1$ root outside unit circle.

4 a) MA(2) $Y_t = Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2}$

b) AR(2) $Y_t - \phi_1 Y_{t-1} - \phi_2 Y_{t-2} = Z_t$

c) MA(1) $Y_t = Z_t + \theta_1 Z_{t-1}$

d) AR(1) $Y_t - \phi_1 Y_{t-1} = Z_t$