CALIFORNIA STATE UNIVERSITY, HAYWARD
DEPARTMENT of Statistics

Statistics 4870/6870 Bayesian Statistics
Summer 2007

Take-home Midterm

Directions: This is an exam, do your own work. You may discuss the exam with the instructor of the class not with other students in the class.

1. (a) (Stat. 4870) For the California death penalty opinion poll use the Bolstad library and also use the betabinimomal.odc WinBUGS program to estimate the parameter θ. Suppose the data is $x = 650$ and the sample size is $n = 1000$.

   i. Using a Uniform(0,1) prior, produce the posterior estimate of θ.

   ii. Using the prior from the in-class midterm, produce the posterior estimate of θ.

   iii. Is there any difference between the posterior estimates of θ using the two priors?

   iv. What is the posterior probability that $\theta > 0.5$? (Hint: The step($e[i]$) function in WinBUGS counts how main times in the Gibbs Sampler that the simulated value of $e[i]$ is greater than 0. So add the line of code,

   ```
   theta.g5 <- step(theta-0.5)
   ```

   to the program and monitor the node theta.g5.)

   (b) (Stat. 6870) For the HIV/AIDS problem on the in-class midterm use the following WinBUGS program to estimate the vector of parameter $\pi = (\pi_1, \pi_2, \pi_3)$. Suppose the data vector is (7,16,977).

   i. What is the distribution of $\mathbf{X} = (X_1, X_2, X_3)$? (Hint: See page .)

   ii. What is a conjugate prior on the vector π? (Hint: See page .)

   iii. Using the following WinBUGS program estimate the posterior distributions of the π's. Use a prior that puts weight 1% on each of the first 2 categories and 98% of the prior weight on the disease free category.

   ```
   model;
   {
       x[1:3] ~ dmulti(p[1:3], n)
       p[1:3] ~ ddirch(alpha[1:3])
       p21 <- step(p[2]-p[1])
   }

   data

   list(x = c(7,16,977), n = 1000, alpha=c(1,1,98))

   inits

   list(p=c(.1,.1,.8))

   list(p=c(.8,.1,.1))

   list(p=c(.1,.8,.1))
   ```

   iv. What is the posterior probability that $\pi_2 - \pi_1 > 0$? (Hint: The step() function in WinBUGS counts how main times in the Gibbs Sampler that the simulated value of π_2 is greater than π_1.)

2. (Stat. 4870/6870) An engineer takes a sample of 5 steel beams from a batch and measures the amount of sag under the standard load. The amounts in mm are: 5.19, 4.72, 4.81, 4.87, and 4.88. It is assumed that the sag is $normal(\mu, \sigma)$ where the standard deviation is $\sigma = 0.25$ is known.

   (a) Use a $normal(5, .5^2)$ prior for μ. Find the posterior distribution using the Bolstad library or WinBUGS.