Homework 2


```
### Ch. 8 Exercise 2

mu <- 0.55

alpha <- 1:2000
beta <- alpha/mu - alpha

# Vector of probabilites for interval (0.51, 0.59)

prob <- pbeta(0.59, alpha, beta) - pbeta(0.51, alpha, beta)
prob.err <- abs(0.95 - prob)  # Errors for the probabilies

# Results: Traget parameter values
t.alpha <- alpha[prob.err==min(prob.err)]
t.beta <- round(t.alpha/mu - t.alpha)

t.alpha
t.beta

# Checking: Achieved mean and probability

a.mean <- t.alpha/(t.alpha + t.beta)
a.mean
a.prob <- pbeta(0.59, t.alpha, t.beta) - pbeta(0.51, t.alpha, t.beta)
a.prob

# Plot of the beta prior with the selected parameter values

x <- seq(0,1,0.001)
y <- dbeta(x,t.alpha,t.beta)

plot(x,y,type='l')
```

(a) $\mu = \frac{\alpha}{\alpha+\beta}$

$\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$

$\mu = E[\pi] = .55$

$P(.51 < \pi < .59) \approx .95$

$P(\mu - 2\sigma < \pi < \mu + 2\sigma) \approx .95$

$\Rightarrow \sigma \approx .02$

So $\frac{\alpha}{\alpha+\beta} = .55$

$.45\alpha = .55\beta$

$\beta = .818\alpha$

$\sqrt{\frac{.818\alpha^2}{(1.818\alpha)^2(1.818\alpha+1)}} = .02$

$\frac{.818\alpha^2}{6.6\alpha^3 + 3.3\alpha^2} = .0004$

$.818 = .0004(6.6\alpha + 3.3)$

```
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>
> alpha <- 1:2000
> beta <- alpha/mu - alpha
>
> # Vector of probabilites for interval (0.51, 0.59)
>
> prob <- pbeta(0.59, alpha, beta) - pbeta(0.51, alpha, beta)
> prob.err <- abs(0.95 - prob)  # Errors for the probabilies
>
> # Results: Traget parameter values
> t.alpha <- alpha[prob.err==min(prob.err)]
> t.beta <- round(t.alpha/mu - t.alpha)
>
> t.alpha
[1] 326
> t.beta
[1] 267
>
> # Checking: Achieved mean and probability
>
> a.mean <- t.alpha/(t.alpha + t.beta)
> a.mean
[1] 0.549747
> a.prob <- pbeta(0.59, t.alpha, t.beta) - pbeta(0.51, t.alpha, t.beta)
> a.prob
[1] 0.9500065
>
> # Plot of the beta prior with the selected parameter values
>
> x <- seq(0,1,0.001)
> y <- dbeta(x,t.alpha,t.beta)
>
> plot(x,y,type='l')
```
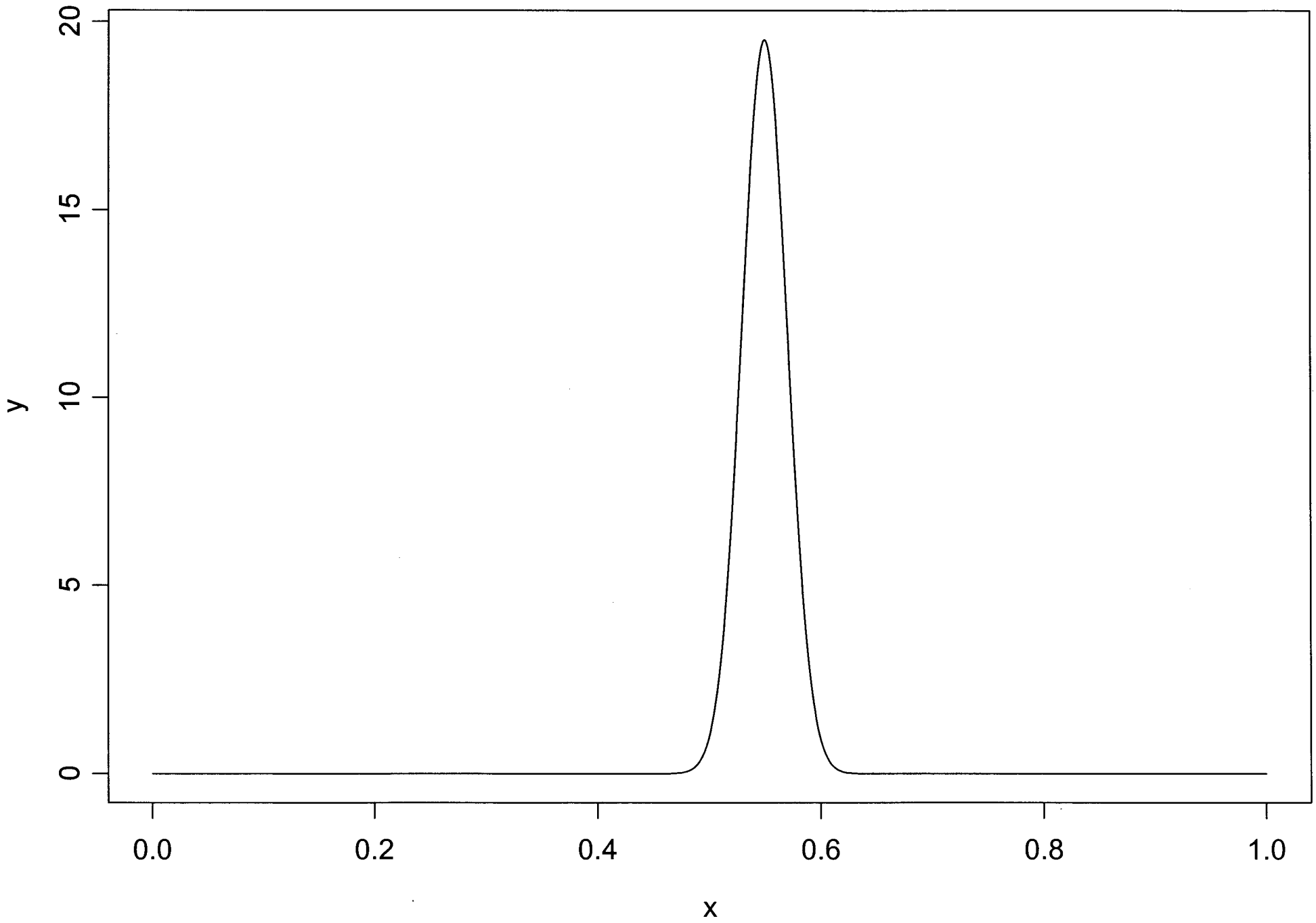
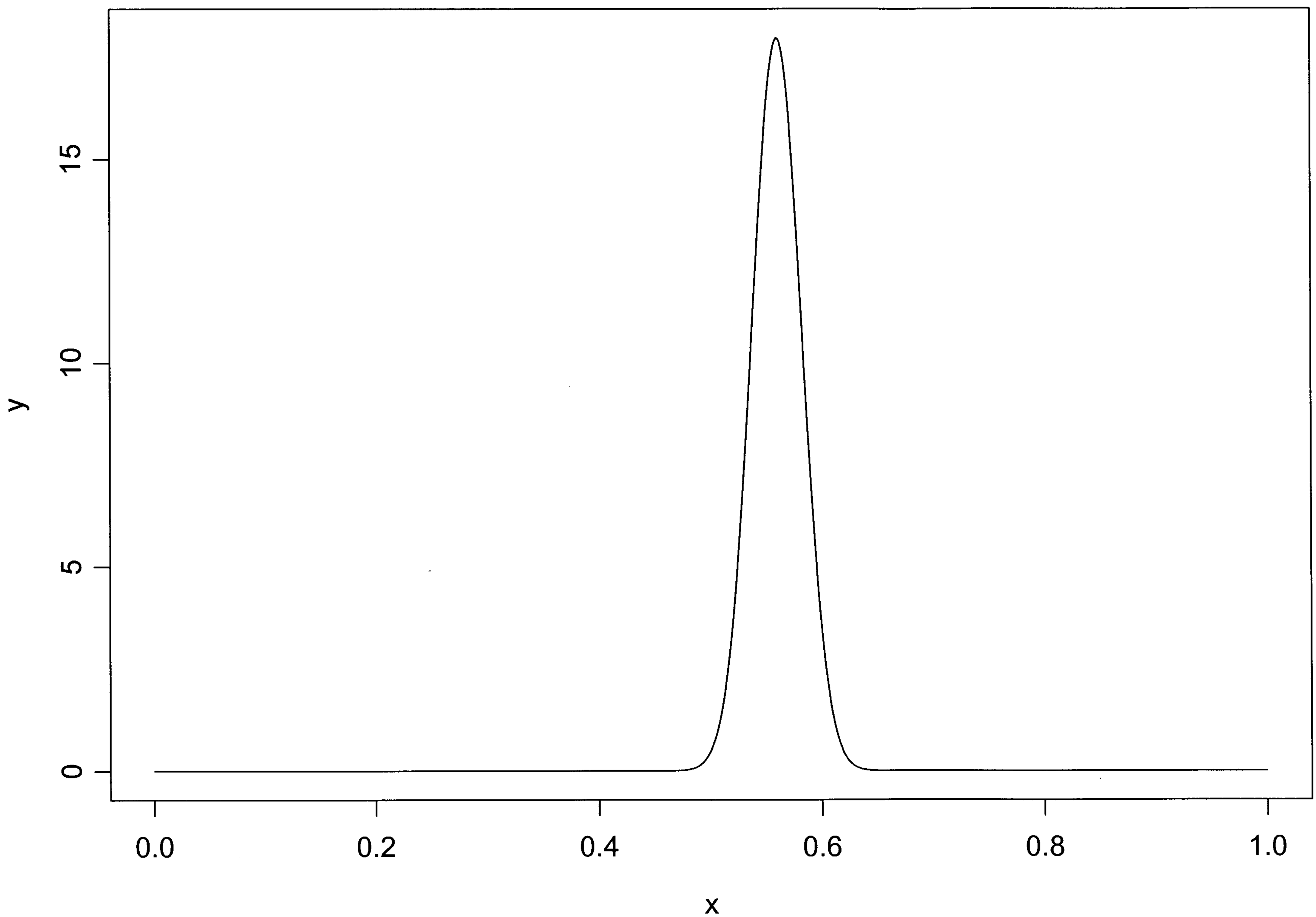


20
15
10
5
0
y
0.0
0.2
0.4
0.6
0.8
1.0
x

```
### Ch. 8 Exercise 2

mu <- 0.56

alpha <- 1:2000
beta <- alpha/mu - alpha

# Vector of probabilites for interval (0.51, 0.59)

prob <- pbeta(0.59, alpha, beta) - pbeta(0.51, alpha, beta)
prob.err <- abs(0.90 - prob)  # Errors for the probabilies

# Results: Traget parameter values
t.alpha <- alpha[prob.err==min(prob.err)]
t.beta <- round(t.alpha/mu - t.alpha)

t.alpha
t.beta

# Checking: Achieved mean and probability

a.mean <- t.alpha/(t.alpha + t.beta)
a.mean
a.prob <- pbeta(0.59, t.alpha, t.beta) - pbeta(0.51, t.alpha, t.beta)
a.prob

# Plot of the beta prior with the selected parameter values

x <- seq(0,1,0.001)
y <- dbeta(x,t.alpha,t.beta)

plot(x,y,type='l')
```

```
> ### Ch. 8 Exercise 2
>
> mu <- 0.56
>
> alpha <- 1:2000
> beta <- alpha/mu - alpha
>
> # Vector of probabilites for interval (0.51, 0.59)
>
> prob <- pbeta(0.59, alpha, beta) - pbeta(0.51, alpha, beta)
> prob.err <- abs(0.90 - prob)  # Errors for the probabilies
>
> # Results: Traget parameter values
> t.alpha <- alpha[prob.err==min(prob.err)]
> t.beta <- round(t.alpha/mu - t.alpha)
>
> t.alpha
[1] 280
> t.beta
[1] 220
>
> # Checking: Achieved mean and probability
>
> a.mean <- t.alpha/(t.alpha + t.beta)
> a.mean
[1] 0.56
> a.prob <- pbeta(0.59, t.alpha, t.beta) - pbeta(0.51, t.alpha, t.beta)
> a.prob
[1] 0.8998249
>
> # Plot of the beta prior with the selected parameter values
>
> x <- seq(0,1,0.001)
> y <- dbeta(x,t.alpha,t.beta)
>
> plot(x,y,type='l')
```


y
0
5
10
15
0.0
0.2
0.4
0.6
0.8
1.0
x