

Bayes' Examples

1. A binary communications system consists of a transmitter that sends 0s and 1s to a receiver over a communication channel. Sometimes errors occur, so that when a 1 is sent a 0 is received, and vice-versa. The probabilities of sending 0s and 1s are p_0 and p_1 respectively. The probability of receiving a 0 when 1 is sent and receiving a 1 when a 0 is sent is p .
 - (a) Develop an expression for $P(R_1)$, the probability of receiving 1s.
 - (b) Develop an expression for $P(R_0)$, the probability of receiving 0s.
 - (c) Find expressions for $P(S_1|R_1)$, i.e., given 1 is received, what is the probability that 1 was sent.
 - (d) Find expressions for $P(S_0|R_0)$, i.e., given 0 was received what is the probability that 0 was sent.
 - (e) Find an expression for the probability of an error in the system.
2. Suppose a factory has two machines A and B that make 60% and 40% of the total production, respectively. Of their output, machine A produces 3% and machine B produces 5% defective items.
 - (a) Find the probability that factory produces a defective part.
 - (b) Find the probability that a given defective part was produced by machine B .
 - (c) Find the probability that a given defective part was produced by machine A .
3. Suppose a test for diagnosing heart disease has a 0.90 probability of positively identifying the disease D when it is present. Suppose the test wrongly positively identifies the disease with probability 0.02 when the disease is not present. From statistical data it is known that 5 of 1000 people in the population have the disease in a certain population. An individual is randomly chosen from this population and is given the test. Calculate the probability that
 - (a) the test is positive, $P(+)$.
 - (b) the individual actually suffers from the disease D if the test turns out to be positive, $P(D|+)$.
 - (c) the individual actually does not suffer from the disease D^c if the test turns out to be positive, $P(D^c|+)$.
 - (d) Is the result for $P(D^c|+)$ surprising? Explain.