Bayesian Deconvolution of Seismic Array Data for Ripple-Fired Explosions

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Treaties:

1. 1963 Limited Nuclear Test Ban Treaty (LTBT)
2. 1968 Non-Proliferation of Nuclear Weapons Treaty (NPT)
3. 1974 Threshold Test Ban Treaty (TTBT)
4. 1976 Peaceful Nuclear Explosions Treaty (PNET)
5. 1996 Comprehensive Test Ban Treaty (CTBT)

Background:

Much of the focus in the past has been on distinguishing possible nuclear explosions from earthquakes.

Currently, since the testing treaties have put limitations on the permissible sizes of the nuclear explosions, other smaller seismic events such as industrial mining explosions have become of interest in the discrimination problem.

The work presented here is related to distinguishing low-level nuclear explosions from ripple-fired mining explosions that are on the same seismic level.

The Problem:

- Monitoring seismic events at Regional Distances for low-level nuclear tests.
- Other seismic sources need to be ruled out.
  - Ripple-Fired Mining Explosions

Ripple-Fired Explosions:

This is a mining technique in which explosions of single devices (or groups of devices) are detonated in succession.

Monitoring:

Arrays of receivers are put in place at Regional Distances and seismic data is continually collected. Seismic disturbances that are above the baseline noise of the area are investigated.

Model:

$$y_k(t) = s_k(t) + \sum_{j=1}^{m} a_j s_k(t-j) + \varepsilon_k(t)$$

Amplitudes are distributed according to a random Bernoulli-Gaussian model. (ref. Cheng, Chen and Li 1996)

$$p(a_j|\eta) \sim (1-\eta)I(a_j = 0) + \eta TN(\mu_\alpha, \sigma_\alpha^2)I(a_j > 0)$$

Signal and path effects follow an $AR(3)$ model. (ref. Dargahi-Noubary 1995, Tjøstheim 1975)

$$s_k(t) + \phi_1 s_k(t-1) + ... + \phi_p s_k(t-p) = e_k(t)$$

$e_k(t)$ i.i.d $N(0, \sigma^2)$ and define the precision $\tau = \frac{1}{\sigma^2}$.

$\varepsilon_k(t)$ i.i.d. $N(0, c\sigma^2)$, $c = 1/SNR$, $c \geq 0$ is fixed.

Truncated Normal:

$$p(a_j) = c^{-1}(\mu_\alpha, \sigma_\alpha^2) \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left\{-\frac{(a_j - \mu_\alpha)^2}{\sigma_\alpha^2}\right\} I(a_j > 0)$$

where

$$c^{-1}(\mu_\alpha, \sigma_\alpha^2) = \int_0^\infty \frac{1}{\sqrt{2\pi}\sigma_\alpha} \exp\left\{-\frac{(x - \mu_\alpha)^2}{\sigma_\alpha^2}\right\}$$

Bayesian Statistics:

Model:

$$p(\mathbf{Y}|\mathbf{\Theta})$$

Prior distribution:

$$p(\mathbf{\Theta})$$

Joint distribution:

$$p(\mathbf{Y}, \mathbf{\Theta}) = p(\mathbf{Y}|\mathbf{\Theta})p(\mathbf{\Theta})$$

Posterior distribution:

$$\begin{aligned} p(\mathbf{\Theta}|\mathbf{Y}) &= \frac{p(\mathbf{Y}|\mathbf{\Theta})p(\mathbf{\Theta})}{\int p(\mathbf{Y}|\mathbf{\Theta})p(\mathbf{\Theta})d\mathbf{\Theta}} \\ &\propto p(\mathbf{Y}|\mathbf{\Theta})p(\mathbf{\Theta}) \end{aligned}$$

Gibbs Sampler: (ref. Gelfand and Smith 1990)

$\mathbf{\Theta} = (\theta_1, \theta_2)$

$p(\mathbf{\Theta}|\mathbf{Y}) = p((\theta_1, \theta_2)|\mathbf{Y})$

Given $\theta_1^{(0)}$ and $\theta_2^{(0)}$ for $h = 1, ..., Reps$

1. Sample $\theta_1^{(h)}$ from $p(\theta_1|\mathbf{Y}, \theta_2^{(h-1)})$
2. Sample $\theta_2^{(h)}$ from $p(\theta_2|\mathbf{Y}, \theta_1^{(h)})$
3. Set $h = h + 1$ and go to 1.

$$(\theta_1^{(BurnIn+1)}, \theta_2^{(BurnIn+1)}), ..., (\theta_1^{(Reps)}, \theta_2^{(Reps)})$$

$\mathbf{\Theta}^{(1)}, ..., \mathbf{\Theta}^{(Reps)}$ are realizations of a stationary Markov Chain, with transition probability from $\mathbf{\Theta}^{(h-1)}$ to $\mathbf{\Theta}^{(h)}$,

$$T(\mathbf{\Theta}^{(h-1)}, \mathbf{\Theta}^{(h)}) = p(\theta_1|\mathbf{Y}, \theta_2^{(h-1)})p(\theta_2|\mathbf{Y}, \theta_1^{(h)})$$

By Ergodic theory, we can calculate estimates of say θ_1 by

$$\frac{1}{Reps}\sum_h \theta_1^{(i)} \xrightarrow{a.s.} E[\theta_1|\mathbf{Y}],$$

$$Reps \to \infty.$$

Priors:

$$\eta \sim BETA(\beta_1, \beta_2)$$

$$\boldsymbol{\phi} \sim N_p(\boldsymbol{\phi}_0, \Sigma_0)$$

$$\tau \sim GAMMA(\gamma_1, \gamma_2)$$

The Model:

$$y_k(t) = s_k(t) + \sum_{j=1}^{m} a_j s_k(t-j) + \varepsilon_k(t)$$

The parameter set:

$\boldsymbol{\Theta} = \{\eta, \mathbf{a}, \boldsymbol{\phi}, \tau, \mathbf{S}\}$

Hyperparamters:

$\beta_1, \beta_2, \mu_\alpha, \sigma_\alpha^2, \boldsymbol{\phi}_0, \Sigma_0, \gamma_1, \gamma_2$

Fixed parameters:

c, m

Likelihood:

$\mathbf{Y} = [\mathbf{y}_1, ..., \mathbf{y}_q]'$

$\mathbf{y}_k = [y_k(1), ..., y_k(n)]'$

$$\begin{aligned} p(\mathbf{Y}|\mathbf{\Theta}) &= \prod_{k=1}^{q} p(\mathbf{y}_k|\mathbf{\Theta}) \\ &= \prod_{k=1}^{q} (2\pi)^{-n/2} \left(\frac{\tau}{c}\right)^{n/2} \exp\left\{-\frac{\tau}{2c}\sum_t \varepsilon_k^2(t)\right\} \end{aligned}$$

Overall Prior:

$$\begin{aligned} p(\boldsymbol{\Theta}) &= p(\eta, \mathbf{a}, \boldsymbol{\phi}, \tau, \mathbf{S}) \\ &= p(\eta)p(\mathbf{a}|\eta)p(\boldsymbol{\phi})p(\tau)p(\mathbf{S}|\boldsymbol{\phi}, \tau) \end{aligned}$$

Joint Density:

$$p(\mathbf{Y}, \boldsymbol{\Theta}) = p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})$$

Joint Posterior:

$$p(\boldsymbol{\Theta}|\mathbf{Y}) \propto p(\mathbf{Y}|\boldsymbol{\Theta})p(\boldsymbol{\Theta})$$

Conditional Marginal Posterior Distributions:

$$p(\eta|\mathbf{Y}, rest) \sim beta(\beta_1^*, \beta_2^*)$$

For fixed $j = 1, ..., m$

$$p(a_j|\mathbf{Y}, rest) \sim (1 - \eta_j)I(a_j = 0) + \eta_j TN(\mu_{a_j}, \sigma_{a_j}^2)I(a_j > 0)$$

$$p(\boldsymbol{\phi}|\mathbf{Y}, rest) \sim N_p(\boldsymbol{\phi}_*, \Sigma_*)$$

$$p(\tau|\mathbf{Y}, rest) \sim gamma(\gamma_1^*, \gamma_2^*)$$

For fixed $i = 1, ..., n$ and $k = 1, ..., q$

$$p(s_k(i)|\mathbf{Y}, rest) \sim N(\mu_{s_k(i)}, \sigma_{s_k(i)}^2)$$

$$\eta|\mathbf{Y}, rest \sim beta(\beta_1^*, \beta_2^*)$$

$$\beta_1^* = m - n_a + \beta_1,$$

$$\beta_2^* = n_a + \beta_2$$

$a_j|\mathbf{Y}, rest \sim$ Bernoulli-Gaussian

$$\mu_{a_j} = \sigma^2_{a_j}\left[\frac{\mu_\alpha}{\sigma^2_\alpha} + \left(\frac{\tau}{c}\right)\sum_k\sum_t \varepsilon^*_k(t[-j])s_k(t-j)\right]$$

$$\sigma^{-2}_{a_j} = \frac{1}{\sigma_\alpha} + \left(\frac{\tau}{c}\right)\sum_k\sum_t s^2_k(t-j)$$

$$\eta_j = \frac{\eta}{\eta + (1-\eta)\left[\frac{c(\mu_{a_j},\sigma^2_{a_j})}{c(\mu_\alpha,\sigma^2_\alpha)}\right]\left(\frac{\sigma_{a_j}}{\sigma_\alpha}\right)\exp\left\{\frac{1}{2}\left[\frac{\mu^2_{a_j}}{\sigma^2_{a_j}} - \frac{\mu^2_\alpha}{\sigma^2_\alpha}\right]\right\}}$$

$$\boldsymbol{\phi}|\mathbf{Y}, rest \sim N_p(\boldsymbol{\phi}_*, \Sigma_*)$$

$$\boldsymbol{\phi}_* = \Sigma_* \left[-\tau \sum_k \sum_t s_k(t)\tilde{\mathbf{s}}_k(t) + \Sigma_0^{-1}\boldsymbol{\phi}_0 \right]$$

$$\Sigma_*^{-1} = \tau \sum_k \sum_t \tilde{\mathbf{s}}_k(t)\tilde{\mathbf{s}}_k'(t) + \Sigma_0^{-1}$$

where

$$\tilde{\mathbf{s}}_k(t) = [s_k(t-1), ..., s_k(t-p)]'$$

$$\tau|\mathbf{Y}, rest \sim gamma(\gamma_1^*, \gamma_2^*)$$

$$\gamma_1^* = q(2n - l - p)/2 + \gamma_1$$

$$\gamma_2^* = \frac{1}{2c}\sum_k\sum_t \varepsilon_k^2(t) + \frac{1}{2}\sum_k\sum_t e_k^2(t)$$

$s_k(t)|\mathbf{Y}, rest \sim \text{Normal}$

$$\mu_{s_k(i)} = \sigma^2_{s_k(i)} \left[\left(\frac{\tau}{c}\right) \sum_t a_{t-i} \varepsilon'(t[-i]) - \tau \sum_t \phi_{t-i} e'_k(t[-i]) \right]$$

$$\sigma^{-2}_{s_k(i)} = \left(\frac{\tau}{c}\right) \sum_t a^2_{t-i} + \tau \sum_t \phi^2_{t-i}$$

Steps To Perform The Gibbs Sampler:

Given the initial values:

$$\left\{\eta^{(0)}, \mathbf{a}^{(0)}, \boldsymbol{\phi}^{(0)}, \tau^{(0)}, \mathbf{S}^{(0)}\right\}$$

for $h = 1$ to Reps:

1. Sample $\eta^{(h)}$ from a *beta*.
2. Sample $a_j^{(h)}$, $j = 1, ..., m$, from a Bernoulli-Gaussian.
3. Sample $\boldsymbol{\phi}^{(h)}$ from a p-variate *normal*.
4. Sample $\tau^{(h)}$ from a *gamma*.
5. Sample $s_k(i)^{(h)}$, $i = 1, ..., n$, $k = 1, ..., q$, from a *normal*.
6. Set $h = h + 1$ and go to Step 1.

Conclusions and Further Work:

1. one
2. two

References:

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