The Poisson Dispersion Test.

The two key assumptions underlying the Poisson distribution are that

1. The rate is constant.
2. The counts in one interval of time or space are independent of the counts in disjoint intervals.

$X_1, \dots, X_n \sim Poi(\lambda)$?

$H_0: X_i \sim Poi(\lambda)$ $\quad H_0: \lambda_1 = \dots = \lambda_n$

$H_1: X_i \sim Poi(\lambda_i)$ $\quad H_1: \lambda_i$'s not all equal

$i = 1, \dots, n.$

under Ω_0 $\quad \hat{\lambda} = \bar{X}$

under Ω $\quad \hat{\lambda}_i = x_i$

$$\Lambda = \frac{\max_{\Omega_0} L(\lambda)}{\max_{\Omega} L(\lambda_1, \dots, \lambda_n)} = \frac{\prod_{i=1}^n \hat{\lambda}^{x_i} e^{-\hat{\lambda}} / x_i!}{\prod_{i=1}^n \hat{\lambda}_i^{x_i} e^{-\hat{\lambda}_i} / \cancel{x_i}!}$$

$$= \prod^n \left(\frac{\bar{X}}{X_i}\right)^{X_i} e^{X_i - \bar{X}}$$

$$-2\log\Lambda = -2 \sum_{i=1}^n \left[x_i \log\left(\frac{\bar{x}}{x_i}\right) + (x_i - \bar{x}) \right]$$

$$= 2 \sum_{i=1}^n x_i \log\left(\frac{x_i}{\bar{x}}\right)$$

using Taylor series approximation

$$-2\log\Lambda \approx \frac{1}{\bar{X}}\sum_{i=1}^{n}(X_i-\bar{X})^2 = \frac{n\hat{\sigma}^2}{\bar{X}}$$

Recall $\frac{(n-1)s^2}{\sigma^2} = \frac{\Sigma(X_i-\bar{X})^2}{\sigma^2}$

If $X_1,\dots,X_n \sim Poi(\lambda)$ then $E[X]=V(X)=\lambda$

under Ω $\quad \lambda_1,\dots,\lambda_n \quad \dim\Omega = n.$

under Ω_0 $\quad \lambda \quad \dim\Omega = 1$

$df = n-1$

Reject H_0 if $-2\log\Lambda > \chi^2_{n-1,\alpha}$

Ex. $X_1,\dots,X_n$ rpois (n,λ) $\quad \bar{X}, S^2$ equal

Ex. $X_1,\dots,X_n$ rnegbin (n, r, p) $\quad \bar{X}, S^2$ not.