CALIFORNIA STATE UNIVERSITY, EAST BAY
STATISTICS DEPARTMENT

Statistics 6502 Mathematical Statistics
Spring 2013

Quiz II

1. **(30 points)** Suppose that $X \sim$ Negative Binomial(r_1, p_1) and $Y \sim$ Negative Bionomial(r_2, p_2). Recall the Negative Binomial distribution.

$$f(x|p) \quad = \quad \binom{x-1}{r-1}(1-p)^r p^{x-r} \qquad x = r, r+1, ...$$

Assume that X and Y independent, we wish to test whether the proportions are equal, $H_0 : p_1 = p_2 = p$ versus $H_1 : p_1 \neq p_2$, where p is unknown.

   (a) The parameter space $\Omega = (0,1) \times (0,1) = \{(p_1, p_2)|\ 0 < p_1 < 1, 0 < p_2 < 1\}$, and the subset corresponding to H_0 is $\Omega_0 = \{(p_1, p_2)|\ 0 < p_1 = p_2 < 1\}$. Sketch and label these sets on one graph.
   (b) Find the MLE's of p_1 and p_2 over Ω.
   (c) Find the MLE of p over Ω_0.
   (d) Find the GLR statistic. Note that the test statistic does not simplify much, but can be computed easily.
   (e) Using the appropriate large sample approximation, specify the GLR test for the significance level α.
   (f) What R code could be used to compute the critical value of test?

2. **(40 points)** Let X have one of the following four distributions:

x	$f_0(x)$	$f_1(x)$	$f_2(x)$	$f_3(x)$
x_1	0.2	0.5	0.3	0
x_2	0.3	0.1	0.0	0
x_3	0.1	0.2	0.4	0.5
x_4	0.4	0.2	0.3	0.5

Determine the G.L.R. test $\phi(x)$ for testing the following simple null hypothesis versus the composite alternative hypothesis

$$H_0 : X \sim f_0(x)$$

$$H_1 : X \sim f_1(x) \quad or \quad X \sim f_2(x) \quad or \quad X \sim f_3(x)$$

at the $\alpha = 0.10$ level? What is the test at the 0.30 level?

3. **(Extra Credit)** You are given a single observation from either the standard normal or the uniform distribution on $(-4, 4)$ distribution.

   (a) Accurately sketch a picture of the null and alternative densities on one set of axes.
   (b) Find the most powerful test of size $\alpha = 0.05$ for

$$H_0 : X \sim Normal(0, 1^2)$$

   versus

$$H_1 : X \sim Uniform(-4, 4).$$

   (c) Calculate the power, π, of the test.
   (d) Calculate the probability of Type II Error, β, for the the test.