Name:

Section: 2p.m. 6p.m.

CALIFORNIA STATE UNIVERSITY, EAST BAY
STATISTICS DEPARTMENT

Statistics 6502 Mathematical Statistics
Winter 2013

Quiz 1

Instructions: This is a closed book and closed notes quiz. You may use a pencil or a pen (blue or black), and write your answers on your own paper. Please write you name on the exam sheet and turn it in with your written solutions on your own paper. Circle the time of the section of the class you are registered for. Please assemble your papers in the order of the problems on the test. The test has a total of 30 points. Use your time appropriately.

1. **(10 points)**

   (a) If you were to choose a p.d.f. to model the amounts of rainfall from different storms, what would it be? Sketch your p.d.f. and give a guess of the value of the parameter(s) for your selected model. Would the distribution change for seeded and unseeded storm clouds in an experiment to determine the effects, if any, of cloud seeding?

2. **(20 points)** Suppose $X_1, X_2, ..., X_n$ is a random sample from

$$f(x|\theta) = (\theta + 2)x^{\theta+1}$$

   for $0 \leq x \leq 1$ and 0 otherwise.

   (a) Find the method of moments estimator (MME) $\tilde{\theta}$ of θ.

   (b) Find the maximum likelihood estimator (MLE) $\hat{\theta}$ of θ.

   (c) (Extra-Credit, due Monday) Find the asymptotic variance of the MLE $\hat{\theta}$.

   (d) (Extra-Credit, due Monday) State the asymptotic distribution of the MLE $\hat{\theta}$.

   (e) (Extra-Credit, due Monday) Give an approximate $100(1-\alpha)\%$ confidence interval for θ based on the the MLE $\hat{\theta}$.

   (f) Using the Inverse CDF Method, give an algorithm for generating a random number from the distribution function $F(x|\theta)$ that corresponds to the density from which the sample is taken. Or name the density $f(x|\theta)$.

   (g) If the sample size were $n = 17$, what might be wrong with using the approximate $100(1-\alpha)\%$ confidence interval from part e)? Describe how you would use the Parametric Bootstrap to get an approximation of the sampling distribution of the MLE $\hat{\theta}$ for a sample of size $n = 17$, an approximation of the standard error $s_{\hat{\theta}}$ of $\hat{\theta}$, and finally an approximate $100(1-\alpha)\%$ bootstrap confidence interval for θ.