Homework 3

Ch. 9

(14) using Thm B

$$\left\{ \underset{\sim}{X} \,\middle|\, \chi^2_{n-1}\left(1-\frac{\alpha}{2}\right) \le \sigma_0^{-2} \sum (x_i - \bar{x})^2 \le \chi^2_{n-1}\left(\frac{\alpha}{2}\right) \right\}.$$

$$\Leftrightarrow \left\{ \underset{\sim}{X} \,\middle|\, \sum (x_i - \bar{x})^2 \ge 26.12 \right\} \cup \left\{ \underset{\sim}{X} \,\middle|\, \sum (x_i - \bar{x})^2 \le 5.63 \right\}.$$

(16)

(16) $X \sim Bin(n, p)$ $\qquad f(x|p) = \binom{n}{x} p^x (1-p)^{n-x} \quad x = 0, 1, \ldots, n$

$H_0: p = .5$
$H_1: p \neq .5$

a) $$\Lambda = \frac{\max_{\Omega_0} f(x|p)}{\max_{\Omega_1} f(x|p)} = \frac{f(x|.5)}{f(x|\hat{p})}$$

$$\hat{p} = \frac{x}{n}.$$

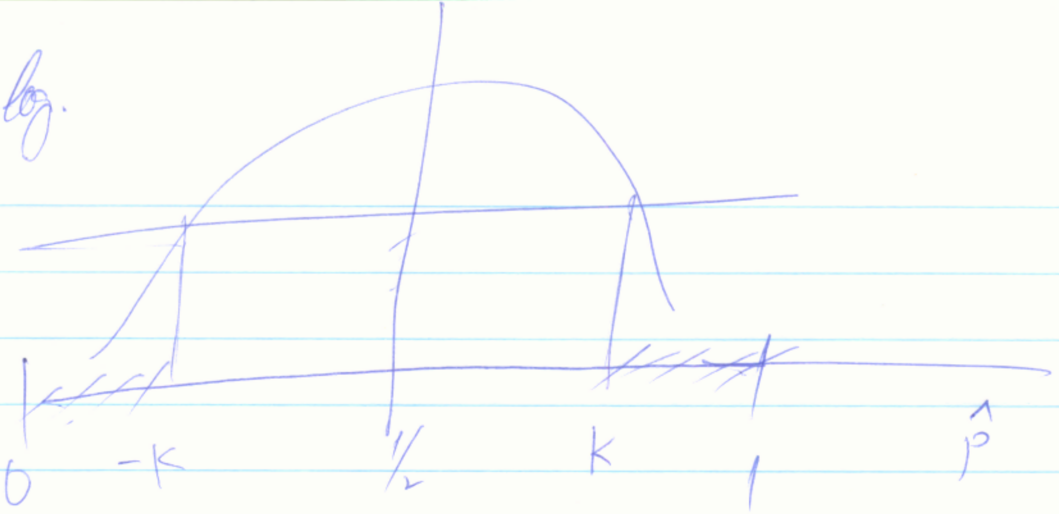
$$= \frac{\cancel{\binom{n}{x}} (.5)^x (1-.5)^{n-x}}{\cancel{\binom{n}{x}} \left(\frac{x}{n}\right)^x \left(1 - \frac{x}{n}\right)^{n-x}} = \frac{\left(\frac{1}{2}\right)^n}{\frac{x^x}{n^x} \frac{(n-x)^{n-x}}{n^{n-x}}}$$

$$= \frac{\left(\frac{n}{2}\right)^n}{x^x (n-x)^{n-x}} = \left(\frac{\frac{n}{2}}{x}\right)^x \left(\frac{\frac{n}{2}}{n-x}\right)^{n-x}$$

$$= \left(\frac{\frac{1}{2}}{\frac{x}{n}}\right)^x \left(\frac{\frac{1}{2}}{1 - \frac{x}{n}}\right)^{n-x} < k$$

take log.

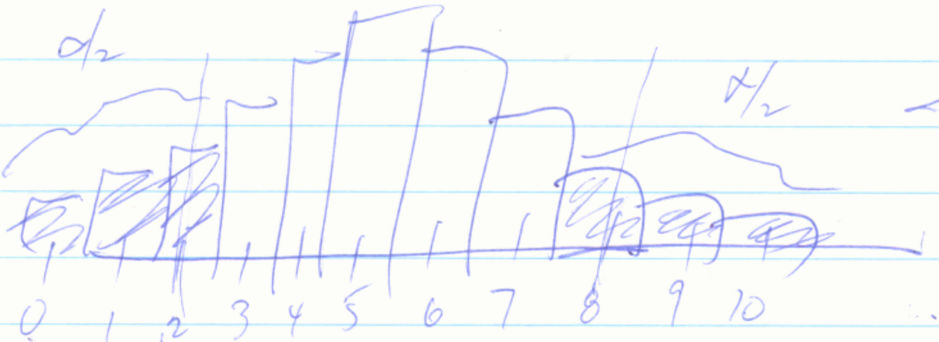
b.)



$$\left|\frac{X}{n} - \frac{1}{2}\right| > K.$$

c.) plot. $\left(\frac{.5}{\hat{p}}\right)^{X} \left(\frac{1-.5}{1-\hat{p}}\right)^{n-X}$

K



~~pbinom~~

c) α = pbinom(k, n p)

+ pbinom((n-k), n, p)

d) k = 2. mu=np sigma=sqrt(np(1-p))

e) α = pnorm((k+.5), mu, sigm) + pnorm((n-k), mu, sigma)

18 a T b. F c. T
d. F e. F f. F

26 $$\chi^2 = \sum_{i=1}^{4} \frac{(X_i - n\,p_i(\hat\theta))^2}{n\,p_i(\hat\theta)}$$ Person's χ^2.

or $$-2\log\Lambda = -2n\sum_{i=1}^{4} \hat p_i \log\left(\frac{p_i(\hat\theta)}{\hat p_i}\right)$$

approximate GLR

$df = (m-1)-1 = m-2 = 2$ since $m=4$

27 same but

$df = (m-1)-1 = m-2 = 1$ since $m=3$

28 $H_0: \theta_i^0 = \frac{1}{12}$ $i = 1, 2, \ldots, 12$ assume same number of days per month

$H_1: \theta_i$

$$\chi^2 = \sum_{i=1}^{12} \frac{(E_i - \theta_i^0)^2}{\theta_i^{0^2}}$$ $E_i = n \cdot \theta_i^0$

$df = (m-1) - 0 = m-1 = 11$

suicide rate seems higher in the summer, except July.

(45) a. simulation   mu = 0, sigma = 1

n = 25 50 100

qqplots   hanging rootgrams

~~Z~~ = rnorm(n, mu, sigma)

b. ~~y~~ = $\cancel{Z}^2$ $\sim \chi^2_n$.

c. $y = \frac{Z}{u}$ u = runif (n, 0, 1)

d) u = runif(n, 0, 1)

lamba = 1

e) e = rexp(n, lamba)

Yes, when n is large.

50 a. qqplot for exp

b. hanging rootgram. compare.

Ch. 11

(4) It is true that when the samples sizes are small, it is very difficult to check the normality of the populations from which the data were sampled. The other issue of sample size may be more important since with the smaller sample size using the normal will be overly optimistic about the variability in the sample mean. With the smaller sample size there is less accuracy in the mean and it is this less accuracy that is included in the t.

(5) It is true that under Ho we would not expect μ_x and μ_y to be exactly equal. However, the ideal under the null is that they are exactly equal. This helps with all of the math and gives a fixed point for comparison with the alternative

(6.) In statistics we always assume random sampling. This type of sampling adds some variability to the estimator $\bar{x}$ and $\bar{y}$, so even if μ_x and μ_y were equal, there is very little chance that $\bar{x} = \bar{y}$. The test takes into account the fact that there is sampling variability in our process. So to reject H_0 the sample means have be be different beyond the expected variability from the sampling method.

(14.) see lecture notes.

(19a) independent two-sample t-test. check normality.

b) nonparametric test.

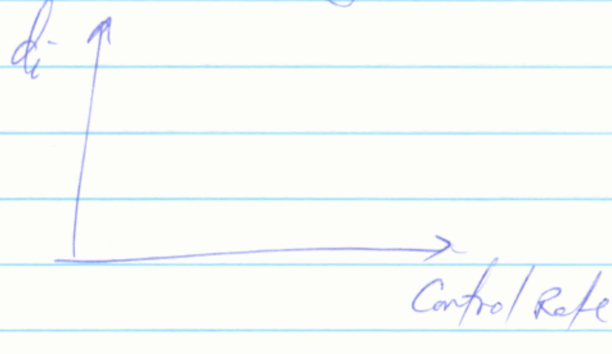
c). depends on the assumptions.

(27.) a) sample(x, without rep) gives a permutation.

do this 1000 times.

b) Mann-Whitney test. ?? nonparametric independent t test.

(37.) a)



b) $\bar{T} - \bar{C}$ $\bar{d}$ s_d

$\bar{d} \pm t \frac{s_d}{\sqrt{n}}$

c). $M_t - M_c$ bootstrap. nonparametric paired t-test.

d) do tests.