

## Exercises

### Maximum Likelihood Estimation

1. Find the m.l.e. of  $\theta$  based on a random sample  $X_1, X_2, \dots, X_n$  from each of the following p.d.f.'s.

(a)

$$\begin{aligned} f(x|\theta) &= \theta x^{\theta-1} & 0 < x < 1, \quad 0 < \theta \\ &= 0 & \text{otherwise} \end{aligned}$$

(b)

$$\begin{aligned} f(x|\theta) &= (\theta + 1) x^{-\theta-2} & 1 < x, \quad 0 < \theta \\ &= 0 & \text{otherwise} \end{aligned}$$

(c)

$$\begin{aligned} f(x|\theta) &= \theta^2 x e^{-\theta x} & 0 < x, \quad 0 < \theta \\ &= 0 & \text{otherwise} \end{aligned}$$

(d)

$$\begin{aligned} f(x|\theta) &= \theta(1 - \theta)^{x-1} & x = 1, 2, \dots, \quad 0 < \theta < 1 \\ &= 0 & \text{otherwise} \end{aligned}$$

2. Find the asymptotic variance of the m.l.e. in each part of question 1.
3. Consider two independent random samples  $X_1, X_2, \dots, X_n \sim N(\mu, \sigma_1^2)$  and  $Y_1, Y_2, \dots, Y_m \sim N(\mu, \sigma_2^2)$ .
- (a) Using the data from the two random samples find the m.l.e. of  $\mu$ ,  $\sigma_1^2$ , and  $\sigma_2^2$ .
- (b) Find the asymptotic variance of  $\mu$ .