Exercises

Maximum Likelihood Estimation

1. Find the m.l.e. of θ based on a random sample $X_1, X_2, ..., X_n$ from each of the following p.d.f.'s.

   (a)

   $$
   \begin{aligned}
   f(x|\theta) &= \theta x^{\theta-1} & 0 < x < 1, \quad 0 < \theta \\
   &= 0 & otherwise
   \end{aligned}
   $$

   (b)

   $$
   \begin{aligned}
   f(x|\theta) &= (\theta+1)x^{-\theta-2} & 1 < x, \quad 0 < \theta \\
   &= 0 & otherwise
   \end{aligned}
   $$

   (c)

   $$
   \begin{aligned}
   f(x|\theta) &= \theta^2 x \exp^{-\theta x} & 0 < x, \quad 0 < \theta \\
   &= 0 & otherwise
   \end{aligned}
   $$

   (d)

   $$
   \begin{aligned}
   f(x|\theta) &= \theta(1-\theta)^{x-1} & x = 1, 2, ..., \quad 0 < \theta < 1 \\
   &= 0 & otherwise
   \end{aligned}
   $$

2. Find the asymptotic variance of the m.l.e. in each part of question 1.

3. Consider two independent random samples $X_1, X_2, ..., X_n \sim N(\mu, \sigma_1^2)$ and $Y_1, Y_2, ..., Y_m \sim N(\mu, \sigma_2^2)$.

   (a) Using the data from the two random samples find the m.l.e. of μ, σ_1^2, and σ_2^2.

   (b) Find the asymptotic variance of μ.