CALIFORNIA STATE UNIVERSITY, EAST BAY
STATISTICS DEPARTMENT

Statistics 6501 Mathematical Statistics
Winter 2011

Take-home Midterm

Instructions: This is the take-home part of the test. This is a test. You are to work on this test alone and you are not to talk with others in the class. This take-home part of the test will be due next week on Monday.

Simulation in R

1. Simulate from the Cauchy distribution 10,000 times, three different ways.
   (a) Generate 10,000 random values from the $Unif(-\frac{\pi}{2}, \frac{\pi}{2})$.
   Let x = atan(u).
   plot(x)
   (b) Generate two vectors of 10,000 random values from the $N(0, 1)$.
   Let w = x/y.
   plot(w)
   (c) Generate two vectors x1 and x2 of 10,000 random values from the $N(0, 1)$.
   Let y1 = x1 + x2 and y2 = x1 - x2.
   plot(y1,y2) Does the plot look uncorrelated?
   Let x.bar = (x1+x2)/2 and s2 = (x1-x2)**2/2
   plot(x.bar, s2) Does the plot look uncorrelated?
   Let t.stat = sqrt(2)*x.bar/sqrt(s2)
   plot(t.stat)

2. Simulate from the general bivariate normal distribution and transform to independence.
   Start by examining the handout BVNsim.R to answer the following questions.
   (a) Make a plot of the $BVN(\mu_1 = 10, mu_2 = 25, \sigma_1^2 = 2^2 = 4, \sigma_2^2 = 3^2 = 9, \rho = -0.4)$.
   (b) Simulate two vectors of $Unif(0, 1)$ random values of length 2,000. Make histograms of each vector of random values and make a scatterplot, one vector on the x-axis and the other on the y-axis.
   (c) Transform the uniform random values to independent standard normal random values using the Box-Muller method. Make histograms of each vector of random values and make a scatterplot, one vector on the x-axis and the other on the y-axis.
   (d) Transform the $BVN(0, 0, 1, 1, 0)$ to $BVN(0, 0, \sigma_1^2, \sigma_2^2, \rho)$. Make histograms of each vector of random values and make a scatterplot, one vector on the x-axis and the other on the y-axis.
   (e) Transform the $BVN(0, 0, \sigma_1^2, \sigma_2^2, \rho)$ to $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. Make histograms of each vector of random values and make a scatterplot, one vector on the x-axis and the other on the y-axis.
   (f) Determine the angle of rotation θ to transform the BVN to independence. Rotate $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$ to $BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, 0)$. Make histograms of each vector of random values and make a scatterplot, one vector on the x-axis and the other on the y-axis.
   (g) Use the R function ipairs( ) in the IDPmisc library to make better scatterplots.
   (h) Use the R function hist2d( ) int he gplots library to make 2 dimensional histograms.

3. Simulate the bivariate p.d.f. of the minimum and maximum sampling from the $N(0, 1)$.
   Simulate the bivariate p.d.f. of the minimum and maximum sampling from the $Unif(0, 1)$.