Simulation Example

Random Number Generator

I will assume that `Random` is a function that returns a random number between 0 and 1. As discussed in class a method for creating such a function is:

```
const C = 25173;
      D = 13849;
      M = 32768;

var Seed : integer;

function Random : real
begin
 Seed := (C * Seed + D) mod M;
 random := Seed / M;
end;
```

the variable Seed must be assigned a randomly selected value between 0 and M-1 at the beginning of the program.

Example

PROBLEM : We want to run a simulation to approximate the probability that in four rolls of a die we observe at least one 6. To do this we need to simulate rolling a die. The statement

```
x := Random;
```

"randomly" assigns a number between 0 and 1 to the variable x. Note that if $0 < x < 1$ then $0 < 6x < 6$. Also since x is equally likely to be any number between 0 and 1, $6x$ is equally likely to be any number between 0 and 6. (This will be proven later in the course.) This means that the integer part of $6x$ is equally likely to be any one of the integers 0, 1, 2, 3, 4, 5. So the statements:

```
x := Random;
y := Trunc(6 * x);
```

would randomly assign one of the integers 0, 1, 2, 3, 4, 5 to the variable y, with each number being equally likely. If we modify this to:

```
x := Random;
y := Trunc(6 * x) + 1;
```

then y would be assigned one of the integers 1, 2,. . . , 6 with each number being equally likely. This is exactly what we want for simulating the roll of a die. We could combine these two statements into one statement:

```
y := Trunc(6 * Random) + 1;
```

The following code simulates rolling a die 4 times. After each roll it checks to see if a six was rolled. If a six is rolled it changes the value of he variable `Yes` to a 1:

```
Yes := 0;
for i := 1 to 4 do
 if ( Trunc(6 * Random) + 1 = 6 ) then Yes := 1;
```

This performs the experiment once. Now we would like to perform the experiment many times and count how many of those times `Yes` turns out to be 1. Say we repeat the experiment 1000 times. We can set up a loop to repeat the above simulation 1000 times and keep count of the number of times `Yes` comes out to be 1 in a variable called `Count`. Then we can approximate the probability of observing at least one six in four rolls by `Count/1000`:

```
program dice;

var i, j, Yes, Count : integer;
   p : real;

begin
Randomize         (* This procedure places a random seed in the random
                     number generator in Turbo Pascal *)
Count := 0;
for j := 1 to 1000 do
 begin
 Yes := 0;
 for i := 1 to 4 do
  if ( Trunc(6 * Random) + 1 = 6 ) then Yes := 1;
 Count := Count + Yes;
 end;
p := count / 1000;
Writeln('The approximate probability is ', p:4:3);
end;
```

In 10 separate runs of this program I get the following results:

```
0.510, 0.516, 0.532, 0.537, 0.507,
0.519, 0.513, 0.502, 0.515, 0.542
```