

## Generating Normal Pseudo-random Numbers

### Box-Muller Method:

How to simulate two independent Normal random variables with mean  $\mu$  and variance  $\sigma^2$ ?

Generating  $U, V$  independent  $UNIF(0, 1)$  random values.

Then

$$W = \cos(2\pi U)\sqrt{-2\ln(V)} \quad \text{and} \quad X = \sin(2\pi U)\sqrt{-2\ln(V)}$$

are such that  $W$  and  $X$  are independent  $Normal(0, 1)$  values.

To get  $Y$  and  $Z$  independent  $N(\mu, \sigma^2)$  calculate

$$Y = \mu + \sigma W \quad \text{and} \quad Z = \mu + \sigma X$$

The following Pascal code implements the Box-Muller Method.

```

FUNCTION Random2 : Real;
{ This function checks if the random number we take the natural log }
{ of is very small. If so, then we set it equal to something small. }
VAR
  x : Real;
BEGIN
  x := Random;
  IF x <= 0.0001 THEN
    Random2 := 0.0001
  ELSE
    Random2 := x;
END;

PROCEDURE Normal(VAR W, X : REAL);
VAR
  T1, T2 : REAL;
BEGIN
  T1 := 2*Pi*Random;
  T2 := sqrt(-2.0*ln(Random2));
  W := cos(T1)*T2;           { Note W and X are independent }
  X := sin(T1)*T2;         { standard Normals.           }
END;

```