A 2-dim rotational transformation. (linear)

$$Y_1 = X_1 \cos\theta + X_2 \sin\theta \qquad \theta \text{ constant}$$
$$Y_2 = -X \sin\theta + X_2 \cos\theta$$

$$\underset{\sim}{Y} = C \underset{\sim}{X} \qquad C = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

means

$$\mu_{Y_1} = E[Y_1] = E[X_1 \cos\theta + X_2 \sin\theta].$$
$$= \mu_{X_1} \cos\theta + \mu_{X_2} \sin\theta. \checkmark$$

$$\mu_{Y_2} = E[Y_2] = E[-X_1 \sin\theta + X_2 \cos\theta].$$
$$= -\mu_{Y_1} \sin\theta + \mu_{X_2} \cos\theta. \checkmark$$

covariance

$$\mathrm{Cov}(Y_1, Y_2) = E[(Y_1 - \mu_{Y_1})(Y_2 - \mu_{Y_2})].$$
$$= E[((X_1 - \mu_{X_1}) \cos\theta + (X_2 - \mu_{X_2}) \sin\theta)$$
$$\cdot(-(X_1 - \mu_{X_1}) \sin\theta + (X_2 - \mu_{X_2}) \cos\theta)]$$
$$= (\sigma_{X_2}^2 - \sigma_{X_1}^2) \sin\theta \cos\theta$$
$$+ \sigma_{X_1 X_2} [\cos^2\theta - \sin^2\theta]$$

$$= \left(\sigma_{X_2}^2 - \sigma_{X_1}^2\right) \tfrac{1}{2} \sin(2\theta)$$

$$+ \sigma_{X_1 X_2} \cos(2\theta)\Big)$$

Recall: double angle formulas

$$\sin 2\theta = 2 \sin\theta \cos\theta$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

For the BVN to be independent, rotate (X_1, X_2) to (Y_1, Y_2) where $Cov(Y_1, Y_2) = 0$.

note: $\sigma_{X_1 X_2} = \rho \sigma_{X_1} \sigma_{X_2}$

$$0 = \left(\sigma_{X_2}^2 - \sigma_{X_1}^2\right) \tfrac{1}{2} \sin(2\theta) + \rho\sigma_{X_1}\sigma_{X_2} \cos(2\theta)$$

$$\left(\sigma_{X_2}^2 - \sigma_{X_1}^2\right) \tfrac{1}{2} \sin(2\theta) = \rho\sigma_{X_1}\sigma_{X_2} \cos(2\theta)$$

$$\tan(2\theta) = \frac{\sin(2\theta)}{\cos(2\theta)} = \frac{2\rho\sigma_{X_1}\sigma_{X_2}}{\sigma_{X_2}^2 - \sigma_{X_1}^2}$$

$$\theta = \frac{1}{2} \tan^{-1}\left(\frac{2\rho\sigma_{X_1}\sigma_{X_2}}{\sigma_{X_1}^2 - \sigma_{X_2}^2}\right)$$

θ is the angle of rotation to transform BVN to independence