CALIFORNIA STATE UNIVERSITY, EAST BAY
STATISTICS DEPARTMENT

Statistics 6502 Mathematical Statistics
Winter 2013

Midterm 1 - Extra Credit

1. **(16 points)** Suppose $X_1, X_2, ..., X_n$ is a random sample from

$$f(x|\theta) = e^{-(x-\theta)}$$

for $x \geq \theta$ and 0 otherwise.

   (a) Find the method of moments estimator (MME) of θ, $\tilde{\theta}$. Is it possible for the MME to underestimate θ?

   (b) Is the MME unbiased? Calculate $E[\tilde{\theta}]$.

   (c) Calculate $Var(\tilde{\theta})$.

   (d) Find the maximum likelihood estimator (MLE) of θ, $\hat{\theta}$. Is it possible for the MLE to underestimate θ?

   (e) Find the probability density of the MLE.

   (f) Is the MLE unbiased? Calculate $E[\hat{\theta}]$.

   (g) Calculate $Var(\hat{\theta})$.

   (h) Suggest a transformation of the MLE that makes it unbiased. Is it possible for the transformed MLE to underestimate θ?

2. **(8 points)** Suppose that certain electronic components have lifetimes that are exponentially distributed: $f(x|\lambda) = \lambda e^{-\lambda x}, x > 0$. Ten new components are put on test and observed for 120 hours. The first three failures were 97, 105 and 110.

   (a) What is the likelihood function of λ? What is the maximum likelihood estimate of λ?

   (b) What is the asymptotic distribution of the maximum likelihood estimate?

   (c) Give an approximate $100(1-\alpha)\%$ confidence interval for λ based on the the maximum likelihood estimator $\hat{\lambda}$.

   (d) Suggest a transformation that might improve the confidence interval from part (c).

3. **(10 points)** The Poisson distribution has been used by traffic engineers as a model for light traffic, based on the rationale that if the rate is approximately constant and the traffic is light (so individual cars move independently of each other), the distribution of counts of cars in a given time interval or space area should be nearly Poisson. The following table shows the number of right turns during 300 3-minute intervals at a specific intersection.

x	Frequency
0	14
1	30
2	36
3	68
4	43
5	43
6	30
7	14
8	10
9	6
10	4
11	1
12	1
13+	0

(a) Fit the Poisson distribution to these data using maximum likelihood estimation. Fit the Poisson model

$$f(x|\lambda) = \frac{\lambda^x}{x!}e^{-\lambda}$$

where $x = 0, 1, ...$ and $\lambda > 0$. Give a numerical answer for the estimate of the parameter in the model.

(b) Comment on the fit by comparing the observed and the expected counts for $x = 1, x = 2$.

(c) Suppose you were not comfortable with the assumed parametric model.

i. Compute the sample mean and sample variance of the data.

ii. Explain how you would use the nonparametric bootstrap to compute confidence intervals for the mean μ and the variance σ^2.

iii. Explain how you would use the nonparametric confidence intervals to check the assumption of the Poisson model.