Project II

Instructions: Complete one of the following suggested probability projects.

1. Read Example E *Bayesian Inference*, Rice page 94 - 95.

   Re-work the problem using a $Beta(\alpha, \beta)$ prior instead of the $Unif(0, 1)$ prior used in the example. Use

   $$f_\Theta(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\theta^{\alpha-1}(1-\theta)^{\beta-1} \tag{1}$$

   for $0 \leq \theta \leq 1$. Determine the posterior density $f_{\Theta|X}(\theta|x)$.

2. Read Example C *Random Walk*, Rice page 140 - 142.

   Write an R program to simulate Brownian Motion for $n = 250$ values. Plot the values. Repeat 10 times. Compare your pictures to the last 250 closing values of a stock traded on the NYSE. Compare your pictures to the last 250 points of a major stock market average such as the DJIA.

3. Read Example B, rice page 154.

   Simulate n = 200 values from the BVN(0, 0, 2, 3, .75). Fit the linear model with only the slope coefficient. Compare the the estimate to the $\rho = 0.75$.

4. Monte Carlo Integration.

   Let $U_1, U_2, ..., U_n$ be independent uniform random values from the interval $[a, b]$. These values have density $f(u) = 1/(b-a)$ on that interval. Then

   $$E[g(U_i)] = \int_a^b g(u)\frac{1}{b-a}\, du \tag{2}$$

   so the original integral

   $$\int_a^b g(x)\, dx \tag{3}$$

   can be approximated by $(b-a)$ times a sample mean of $g(U_i)$.

   Use Monte Carlo integration to estimate the following integrals. Compare with the exact answers, if known.

   (a)

   $$\int_0^1 x\, dx$$

   (b)

   $$\int_1^\pi e^x\, dx$$

(c)

$$\int_0^\infty e^{-x}\ dx$$

(d)

$$\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx$$

(e)

$$\int_0^2 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx$$

(f)

$$\int_0^3 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\ dx$$

(g)

$$\int_0^1 \int_0^1 e^{-(x+y)^2} (x+y)^2\ dx$$

5. If you have another idea for a project, please propose it during office hours for approval.