Limiting Distribution and CLT Problems

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from $Exp(\lambda)$. Let $Y_1 = \min\{X_1, X_2, \ldots, X_n\}$. Find the limiting distribution of Y_1.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from $F(x) = 1 - \frac{1}{x}, x \geq 1$.

   (a) Find the c.d.f. of $Y_1 = \min\{X_1, X_2, \ldots, X_n\}$.

   (b) Let $Z_n = n(1 - F_X(Y_n))$ where $Y_n = \max\{X_1, X_2, \ldots, X_n\}$, find
   $$\lim_{n \to \infty} F_{Z_n}(t).$$

3. Let $X_1, X_2, \ldots, X_n$ be independent $\chi^2(1)$ random variables. Then
   $$Y_n = \sum_{i=1}^{n} X_i \sim \chi^2(n).$$

   The C.L.T. says $Z_n = \frac{Y_n - n}{\sqrt{2n}} \xrightarrow{d} Z \sim N(0,1)$.

   Show that $W_n = \sqrt{2Y_n} - \sqrt{2n-1} \xrightarrow{d} Z \sim N(0,1)$.