Simulation Exercises:

1. (L.L.N) Demonstrate the Law of Large Numbers for the Normal Distribution.
    - Sample n = 1000 standard normal random values and put them in a vector z.
    - Write an S-plus function to compute the cumulative mean.
    - Create a vector x = [1,2,…,n].
    - Compute the cumulative mean vector y from z.
    - Make a convergence plot of y versus x.
    - Approximate the probability that $|y| > \varepsilon$.
    - Hint: Here is the splus code for the cummean function:

```
# cummean function: calculates the cumulative mean of a vector.

cummean <- function(x){

        n <- length(x)

        y <- numeric(n)

        z <- c(1:n)

        y <- cumsum(x)

        y <- y/z

        return(y)

}
```

2. (C.L.T.) Demonstrate the Central Limit Theorem for the Normal and Exponential Distributions.
    - Sample k = 1000 samples of size n = 30 from the *Normal(5,2).*
    - Compute the mean of the k samples.
    - Draw a histogram of these samples.
    - Describe the shape of the histogram.
    - Do the above steps again with the $Exp(\lambda = \frac{1}{3})$.
3. Demonstrate the independence of the sample mean and variance when sampling from the *Normal(5,2).*
    - Sample k = 1000 samples of size n = 30 from the *Normal(5,2).*
    - Compute the sample mean and standard deviation of the k samples.
    - Plot the samples means versus the sample standard deviation.
    - Compute the correlation between the means and standard deviation.
    - Do the above steps again with the $Exp(\lambda = \frac{1}{3})$.