Simulation Exercises

1. (L.L.N.) Demonstrate the Law of Large Numbers for the Normal Distribution.
   - Sample $n = 1000$ Standard Normal random values and put them in a vector z.
   - Write an R function to compute the cumulative mean.
   - Create a vector $\mathbf{x} = (1, 2, ..., n)$.
   - Compute the cumulative mean vector y from z and x.
   - Approximate the probability that $|y| > \epsilon$.
   - Hint: Here is the R code needed for writing the cummean function.

```
cummean = function(w){
        n = length(w)
        y = numeric(n)
        x = c(1:n)
        y = cumsum(w)
        y = y/x
        return(y)
}
```

2. (C.L.T.) Demonstrate the Central Limit Theorem for the Normal and Exponential Distributions.
   - Sample $k = 1000$ samples of size $n = 30$ from the $Normal(5, 2)$.
   - Compute the means for each of the k samples.
   - Draw a histogram of these k sample means.
   - Describe the shape of the histogram.
   - Do the above steps again with the $Exp(\lambda = 1/3)$.

3. Demonstrate the independence of the sample mean and variance when sampling from the $Normal(5, 2)$.
   - Sample $k = 1000$ samples of size $n = 30$ from the $Normal(5, 2)$.
   - Compute the sample means and standard deviations for each of the k samples.
   - Plot the sample means versus the sample standard deviations.
   - Compute the correlation between the sample means and standard deviations.
   - Do the above steps again with the $Exp(\lambda = 1/3)$.