## Homework Set No. 2

Answer the following questions. Please provide a type in copy of the solution.

Q\#1: A protein database contains 10 protein sequences. 4 proteins in this database are homologues and form a protein family:
a) How many different possible ways are there of forming this family.
b) Three other proteins also constitute a family. How many different ways are there of placing the 10 proteins into the two families.

Q\#2: The PFAM profile database contains 2700 domain families:
a) How many different four domain protein are possible.
b) How many four domain proteins are possible in which no two domains in the proteins are homologous to each other.

Q\#3: Consider only standard nucleotides (A,T,G, or C) and standard amino acids (20 of them)
a) How many unique 30 -base long DNA sequences are there?
b) How many unique 10 -residue long peptide sequences are there?
c) Considering that the correspond DNA sequence of a 10 -residue long peptide is 30 nucleotide long, should the two numbers obtained in a.) and b.) be the same? If not, why?

Q\#4: How often an TaqI site would be expected to appear by chance in a random sequence.

Q\# 5: A base calling procedure is very acurate in determining the nucleotides in a DNA sequence, in that it correctly identifies each base with high probability and only rarely misclassifies bases. Let $E_{A}, E_{C}, E_{G}$ and $E_{T}$ be the events that base under study is identified as an $A, C, G$ and $T$ respectively, and let $F_{A}, F_{C}, F_{G}$ and $F_{T}$ be the events that base under study is actually an $A, C, G$ and $T$ respectively. Suppose that prior to any analysis being carried out it is assumed that:

$$
\begin{aligned}
& p\left(F_{A}\right)=p_{A}=0.30 \\
& p\left(F_{G}\right)=p_{G}=0.20 \\
& p\left(F_{C}\right)=p_{C}=0.20 \\
& \left.p_{T}\right)=p_{T}=0.30
\end{aligned}
$$

and the conditional probabilities of (mis) classification of a base, give that its actual type are given by the following tables

|  |  | Is Called As |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | A | C | G | T |
|  | A | 0.900 | 0.025 | 0.025 | 0.050 |
| The Base | C | 0.025 | 0.850 | 0.100 | 0.025 |
|  | G | 0.025 | 0.100 | 0.850 | 0.025 |
|  | T | 0.050 | 0.025 | 0.025 | 0.900 |

so that, for example, from the top row

$$
p\left(E_{A} \mid F_{A}\right)=0.900 \quad p\left(E_{C} \mid F_{A}\right)=p\left(E_{G} \mid F_{A}\right)=0.025 \quad p\left(E_{T} \mid F_{A}\right)=0.05
$$

and so on.
i). Using the Total probability formula, compute the probability that an unknown base under analysis is classified as $i \in\{A, C, G, T\}$, that is, compute

$$
p\left(E_{i}\right)=\sum_{j \in\{A, C, G, T\}} p\left(E_{i} \mid F_{j}\right) p\left(F_{j}\right) \quad \text { for each } i \in\{A, C, G, T\}
$$

ii). Compute, using Bayes Theorem or the conditional probability formula, the conditional probability that a base is actually an $A$, given that is classified as an $A$ i.e., $p\left(F_{A} \mid E_{A}\right)$.

Compute also the three conditional probabilities that a base is actually an $A$, given that it classified as $C$.

Q\#6: Count data from two DNA sequences was collected

|  | Nucleotide |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Sequence | A | C | G | T | Total |
| 1 | 250 | 140 | 180 | 230 | 800 |
| 2 | 320 | 270 | 310 | 300 | 1200 |
| Total | 570 | 410 | 490 | 530 | 2000 |

A test of the null hypothesis $H_{0}$, that the marginal probabilities of the four nucleotides are identical for both sequences, is required.
i). Complete the table of expected counts

$$
e_{i j}=\frac{n_{i .} n_{. j}}{n} \quad i=1, \ldots, r, j=1, \ldots, c
$$

Assuming $H_{0}$ is true, where $n_{i .}$ is the total of the $i^{\text {th }}$ row, $n_{. j}$ is the total of the $j^{\text {th }}$ column, and $n$ is the total number of observations.
ii). Compute the Chi-squared statistic $\chi^{2}$.

Recall that, here, the test statistic is defined as

$$
\chi^{2}=\sum_{i=1}^{2} \sum_{j=1}^{4} \frac{\left(n_{i j}-e_{i j}\right)^{2}}{e_{i j}}
$$

iii). Carry out a test of $H_{0}$ at the significance level of $\alpha=0.01$.

Note: You could verify your result by using Minitab or R. The R-code to solve this problem is:

```
\(x<-c(250,140,180,230,320,270,310,300)\)
data.matrix \(<-\) matrix \((x\), ncol \(=4\), byrow \(=T R U E)\)
\(c<-\) colSums(data.matrix)
\(r<-\operatorname{rowSums}(\) data.matrix)
gt \(<-\operatorname{sum}(\) data.matrix)
\(p<-\operatorname{matrix}(0\), ncol \(=4\), nrow \(=2)\)
for(i in 1:2)
\(\{\) for (j in 1:4)
\(\left.\left\{p[i, j]<-c[j] * r[i] / g t^{2}\right\}\right\}\)
chi.test \(<-\) chisq.test(data.matrix, \(p\) )
```

Here some information about the chi-square functions available in R :
$d c h i s q(x, d f, n c p=0, l o g=F A L S E)$
pchisq $(q, d f, n c p=0$, lower.tail $=T R U E, \log \cdot p=F A L S E)$
$q c h i s q(p, d f, n c p=0$, lower.tail $=T R U E, \log \cdot p=F A L S E)$
$\operatorname{rchisq}(n, d f, n c p=0)$
Arguments:
$x, q$ : vector of quantiles.
$p$ : vector of probabilities.
$n$ : number of observations. If 'length $(n)>1$ ', the length is taken to be the number required.
df: degrees of freedom.
$n c p$ : non-centrality parameter. For 'rnchisq', ' $n c p=0$ ' is the only possible value. $\log , \log . p$ : logical; if TRUE, probabilities $p$ are given as $\log (p)$.
lower.tail: logical; if TRUE (default), probabilities are $P[X \leq x]$, otherwise, $P[X>x]$.

