Statistics 3601 Introductory Statistics for Scientists and Engineers
Winter 2001

Box-Muller Method:

How to simulate two independent Normal random variables with mean μ and variance σ^2.

Generate *U*, *V* independent *UNIF(0,1)* random values.

Then

$$W = \cos(2\pi U)\sqrt{-2\ln(V)} \quad \text{and} \quad X = \sin(2\pi U)\sqrt{-2\ln(V)}$$

are such that *W* and *X* are independent *Normal(0,1)* values.

To get *Y* and *Z* independent $N(\mu, \sigma^2)$ calculate

$$Y = \mu + \sigma W \quad \text{and} \quad Z = \mu + \sigma X \ .$$

The following Pascal code implements the Box-Muller Method.

```
FUNCTION Random2 : Real;
{ This function checks if the random number we take the natural log  }
{ of is very small.  If so, then we set it equal to something small. }
VAR
      x : Real;
BEGIN
      x := Random;
      IF x <= 0.0001 THEN
            Random2 := 0.0001
      ELSE
            Random2 := x;
END;


PROCEDURE Normal(VAR W, X : REAL);
VAR
      T1, T2 : REAL;
BEGIN
      T1 := 2*Pi*Random;
      T2 := sqrt(-2.0*ln(Random2));
      W := cos(T1)*T2;                  { Note W and X are independent }
      X := sin(T1)*T2;                  { standard Normals.            }
END;
```