5.4 **a.** Notice that all of the probabilities are at least 0 and sum to 1

b. Note $F(1,2) = P(Y_1 \le 1, Y_2 \le 2) = 1$. The interpretation of this value is that every child in the experiment either survived or didn't and used either 0, 1 or 2 seatbelts.

5.5 **a.**

$$P(Y_1 \le \frac{1}{2}, Y_2 \le \frac{1}{3}) = \int_0^{1/3}\int_{y_2}^{1/2} 3y_1 \, dy_1 \, dy_2 = \int_0^{1/3} \left[\frac{3}{2}y_1^2\right]_{y_2}^{1/2} dy_2$$

$$= \int_0^{1/3} \left(\frac{3}{8} - \frac{3}{2}y_2^2\right) dy_2 = \left[\frac{3}{8}y_2 - \frac{1}{2}y_2^3\right]_0^{1/3}$$

$$= \frac{1}{8} - \frac{1}{54} \approx .11.$$

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Solution

Note that performing the integration in this order prevented splitting the integral into two parts.

b. $P(Y_2 < Y_1/2) = \int_0^1 \int_0^{y_1/2} (3y_1) \, dy_2 \, dy_1 = \int_0^1 (3y_1) \frac{y_1}{2} \, dy_1 = \left[\frac{1}{2}y_1^3\right]_0^1 = \frac{1}{2}$.

5.20 a. The marginal distributions for Y_1 and Y_2 are given in the margins of the table. That is, the marginal distribution for Y_1 is $P(Y_1 = 0) = .76$ and $P(Y_1 = 1) = .24$ and the marginal distribution for Y_2 is given by $P(Y_2 = 0) = .55$, $P(Y_2 = 1) = .16$ and $P(Y_2 = 2) = .29$.

b. $P(Y_2 - 0|Y_1 = 0) = \frac{P(Y_2=0,Y_1=0)}{P(Y_1=0)} = \frac{.38}{.76} = .5$, $P(Y_2 = 1|Y_1 = 0) = \frac{.14}{.76} = .18$

$P(Y_2 = 2|Y_1 = 0) = \frac{.24}{.76} = .32$.

c. The desired probability $P(Y_1 = 0|Y_2 = 2) = \frac{.38}{.55} = .69$.

5.21 a. $f(y_2) = \int_{y_2}^{1} 3y_1 dy_1 = \frac{3}{2}y_1^2\Big|_{y_2}^{1} = \frac{3}{2} - \frac{3}{2}y_2^2$ for $0 \le y_2 \le 1$.

b. $f(y_1|y_2)$ is defined for $0 \le y_2 < 1$.

c. Note that $f(y_1) = \int_0^{y_1} 3y_1 dy_2 = 3y_1^2$. Then $f(y_2|y_1) = f(y_1,y_2)/\, f(y_1) = 1/y_1$ for $0 \le y_2 \le y_1$. Specific to this problem we consider $f(y_2|y_1 = \frac{3}{4}) = \frac{4}{3}$. The probability in question is $\int_{1/2}^{3/4} \frac{4}{3} dy_2 = 1 - \frac{4}{6} = 1/3$.

5.42 Dependent, for example $P(Y_1 = 0, Y_2 = 0) \neq P(Y_1 = 0)P(Y_2 = 0)$.

5.43 Dependent as the range of y_1 values on which $f(y_1, y_2)$ is defined depends on y_2.
More rigorously recall from exercise 5.21 $f(y_2|y_1) = 1/y_1$ for $0 \leq y_2 \leq y_1$ and $f(y_2) = \frac{3}{2} - \frac{3}{2}y_2$ for $0 \leq y_2 \leq 1$. Thus $f(y_2|y_1) \neq f(y_2)$ for all values for which $f(y_2) > 0$, hence by problem 5.37 Y_1 and Y_2 are dependent.