Practice Problems

3.18 Y = number of bottles with serious flaws

assuming independence

y	0	1	2
$p(y)$	.81	.18	.01

$P(Y=0) = P(\bar{F}\bar{F}) = P(\bar{F})P(\bar{F}) = (.90)^2 = .81$

$P(Y=1) = P(F\bar{F} \text{ or } \bar{F}F)$

$= P(F\bar{F}) + P(\bar{F}F)$

$= P(F)P(\bar{F}) + P(\bar{F})P(F)$

$= 2(.10)(.90) = .18$

$P(Y=2) = P(FF) = P(F)P(F) = (.10)^2 = .01$

So

$\mu = \Sigma y\, p(y) = 0(.81) + 1(.18) + 2(.01) = .20$

$\sigma^2 = E[(Y-\mu)^2] = E[Y^2] - \mu^2 = \Sigma y^2 p(y) - \mu^2$

$= [0^2(.81) + 1^2(.18) + 2^2(.01)] - (.2)^2 = .22 - .04 = .18$

3.30 Y = number of falsified application forms

Then Y is $Bin(n=5, p=.35)$

a) $P(Y \geq 1) = 1 - P(Y=0) = 1 - \binom{5}{0}(.35)^0(.65)^5 = 1 - .116 = .884$

b) $P(Y \geq 2) = 1 - P(Y=0) - P(Y=1)$

$= 1 - .116 - \binom{5}{1}(.35)^1(.65)^4$

$= .884 - .312 = .572$

3.57 Y = The number of the first account containing errors

Then Y is $Geo(p=.9)$

a) $P(Y=3) = (.1)^2(.9) = .009$

b) $P(Y \geq 3) = 1 - P(Y \leq 2) = 1 - P(Y=1) - P(Y=2)$

$= 1 - .9 - (.1)(.9) = .01$

3.90 Y = number of malfunctioning copiers selected.

Then Y is HYP

$$P(Y=y) = \frac{\binom{3}{y}\binom{5}{4-y}}{\binom{8}{4}} \quad \text{for } y = 0, 1, 2, 3$$

a) $$P(Y=0) = \frac{\binom{3}{0}\binom{5}{4}}{\binom{8}{4}} = \frac{5}{70} = \frac{1}{14}$$

b) $$P(Y \geq 1) = 1 - P(Y=0) = 1 - \frac{1}{14} = \frac{13}{14}$$

3.104 Y = number of cars entering the tunnel in a two-minute period.

Then Y has a Poisson($\lambda = 1$)

$$P(Y > 3) = 1 - P(Y \leq 3) = 1 - \sum_{y=0}^{3} \frac{e^{-1} 1^y}{y!}$$

$$= 1 - \frac{e^{-1}}{0!} - \frac{e^{-1}}{1!} - \frac{e^{-1}}{2!} - \frac{e^{-1}}{3!}$$

$$= 1 - \frac{8}{3} e^{-1} = .01899$$