STANFORD UNIVERSITY
STATISTICS DEPARTMENT

Statistics 207 Introduction to Times Series Analysis
Summer 2001

Midterm

Instructions: This is a closed-book and closed-notes exam. You may use 2 (or 3) pages of notes on the exam.

1. Consider a time series $\{x_t\}$ consisting of a quadratic trend and additive independent white noise $\{w_t\}$ with mean 0 and variance σ^2. That is

$$x_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + w_t \tag{1}$$

   where α_0, α_1, and α_2 are unknown parameters.

   (a) Show that $\{x_t\}$ is nonstationary.

   (b) Suggest a transformation of $\{x_t\}$ that is stationary and prove that it is in fact stationary.

2. Identify the following ARMA models and determine if they are causal and/or invertible.

   (a) $(1 - 1.05B + 0.4B^2)x_t = w_t$

   (b) $(1 - 1.05B)x_t = w_t$

   (c) $(1 + 0.8B)x_t = (1 - 0.25B)w_t$

3. Consider the ARIMA(0,1,1) model written as

$$x_t = x_{t-1} + w_t - \theta w_{t-1} \tag{2}$$

   where $|\theta| < 1$.

   (a) Show that the model is invertible and solve for the $\{\pi_j\}$ coefficients.

   (b) Re-write the invertible representation of the model where x_t is equal to past values of time series and white noise.

   (c) Determine the one-step ahead prediction formula by taking the conditional expectation. Note that it can be written in the form

$$\tilde{x}_{n+1} = (1 - \theta)x_n + \theta\tilde{x}_{n-1} \tag{3}$$

   for , $n > 1$.

   (d) Indicate how the prediction formula could be used in practice. In particular indicate where truncation will be need to make the calculations. (Remark: The prediction formula derived is a forecasting method known as exponentially weighted moving averages, (EWMA).)

4. Consider the following pairs of ACF's and PACF's. In each case indicate whether the pairs of diagrams is associated with an AR or MA process, state the order of the process, and in each case write an appropriate model that you think would fit the data well.